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Professor H. H. TURNER, M.A., Sc.D., F.R.S., PRESIDENT, in the Chair.

Henry John Wolverton Brennand, B.A., M.B., Ch.M., F.C.S., Government Statistician's Department, and 203 Macquarie Street, Sydney, New South Wales ;
Albert Edward Garrett, F.R.G.S., 127 Lothair Road, Finsbury Park, N. ;
Joseph Massey Harvey, 77 Cloudesdale Road, Balham, S.W. ;
John George Hatchard, C.S.A. Railway Works, Pretoria, Transvaal, South Africa ;
Rev. Gustavo Heredia, S.J., Stonyhurst College, Blackburn, Lancashire ;
Herbert Kitchin, Marshall Square, Johannesburg, Transvaal, South Africa ;
George Aimer Russell, M.A., B.Sc., 29 Glebe Road, Kilmarnock, Scotland ; and
Frank Herbert Shaw, Ferndale, Gledholt, Huddersfield ;

were balloted for and duly elected Fellows of the Society.

The following Candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Percy Morris, Holmwood, Camborne Road, Sutton, Surrey (proposed by H. F. Newall).

Sixty-five presents were announced as having been received since the last meeting, including, among others :—

Annals of the Royal Observatory, Cape of Good Hope, vol. 11 (S. S. Hough, Heliometer triangulation of the southern circumpolar area) ; Harvard College Observatory Annals, vol. 51 (W. H. Pickering, Photographic Atlas of the Moon) ; Helsingfors Observatory, Catalogue photographique du Ciel, coordonnées rectilignes, &c., tome 4; Rome, Specola Vaticana, Catalogo fotografico stellare coordinati rettilinei &c., vol. 1 (presented by the Observatories).

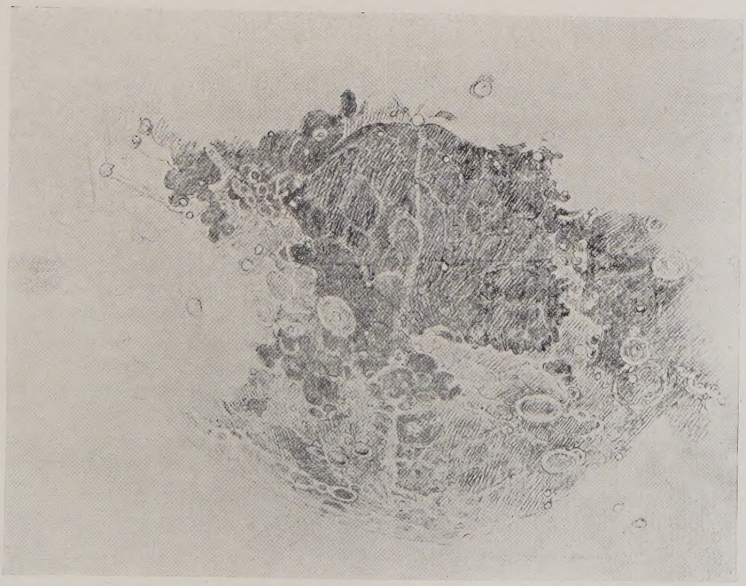
Two Drawings of the Mare Serenitatis by John Russell, R.A., affording some hitherto unpublished evidence as to the appearance of Linné in the year 1788. By Arthur A. Rambaut, F.R.S., Radcliffe Observer.

In the *Monthly Notices* of the Royal Astronomical Society, vol. lvi. p. 3, appears a paper, by the late Mr. Stone, entitled "Note on a Crayon Drawing of the Moon, by John Russell, R.A., at the Radcliffe Observatory, Oxford." In this paper a passing reference is made to a large number of detailed pencil drawings of different portions of the Moon's surface, which are also in the possession of the Radcliffe trustees, and a photographic reproduction of one of them is given in the paper ; but no particulars are afforded with respect to them, Stone's attention being confined to the large picture representing the gibbous Moon, which still hangs in the library of the Radcliffe Observatory, and of which a photographic reproduction is also given in the paper.

Russell's drawings, which were for the most part made on separate sheets of paper, are all contained in a large folio volume bound in russia leather, and were presented to the Radcliffe Observatory by his grandson, Rev. S. H. Russell, B.D., on August 4, 1873. This volume contains 187 separate telescopic studies of limited regions of the lunar surface. They are all carefully mounted and protected from injury by sheets of tissue paper ; and although now more than 100 years old, having with one exception been made between 1785 and 1797, they are still in a very good state of preservation. Many of the drawings have memoranda in Byrom's system of shorthand written upon them, giving in most cases the date on which the drawing was made, but unfortunately this particular is sometimes omitted, and in such cases the only clue to the date of the drawing is afforded by its position in the volume with regard to the other dated drawings and the approximate phase of the Moon indicated by the drawing itself. I have, however, so far been unable to ascertain by whom and on what system the studies were arranged ; and as in some cases the year given is manifestly incorrect, this clue cannot always be relied upon. Such errors can, however, generally be

Fig. 1.

Fig. 2.



DRAWINGS OF THE MARE SERENITATIS, BY JOHN RUSSELL, R.A. 1788.

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corrected without difficulty. As, for instance, when he writes, "1/2 after 7 in the evening 1790 January 19. Full Moon to-morrow at 6 in the morning." The full Moon of January 1790 occurred at 7 P.M. on the 30th, whereas in the following year it occurred at 6 A.M. on the 20th. Whence it is clear that the mistake is of a sort that most of us have made from time to time, especially in the month of January.

From a memorandum left by Professor Rigaud, we find that "Mr. R. had a 6-feet reflector of Dr. Herschell's with which he observed, and also a telescope provided with a micrometer: with this he took all his own measures."

We have no information as to the aperture or focal length of the latter instrument, but are told that it was an achromatic telescope, and that with it most of Russell's observations were made; "for, although Dr. Herschel had provided him with a reflector, Mr. Russell made very little use of it." On the same authority we learn: "Mr. R. always preferred a low power in examining the Moon."

My attention has recently been directed to these drawings by Mr. S. A. Saunder, F.R.A.S., in connexion with the question as to possibility of change in Linné. At his request I have had these drawings examined with a view to ascertaining what evidence they may afford with regard to the condition of this marking in Russell's time. Out of the 187 drawings it is found that only two represent this object. The first of these, which is represented in Plate 3, fig. 1, is particularly interesting.* In the original Linné is somewhat whiter and more conspicuous than it appears in fig. 1, Plate 3. Owing to the age of the paper on which the drawing was made, some effect of contrast has been lost in the process of reproduction. As indicated by the shorthand note inscribed upon it, this drawing represents the appearance of the Mare Serenitatis from 9 to 11 on 1788 April 12. It was thus executed in the same year as Schröter's drawing † of 1788 November 5, and under very nearly the same conditions as to the phase of the Moon. The shorthand note on this drawing runs as follows: "From 9 to 11 on the 12th April, 1788. First quarter to-morrow noon. This was drawn to show the small circular pit situated upon the white line, that it is deeper than the general surface, yet empty, the shadow of the rim bearing no proportion with the hollow centre." The rest of the legend is rather obscure, some of the characters being indistinct; but as the small circular pit referred to is Bessel, not Linné, this is not of so much consequence. Opposite Manilius, and just beyond the terminator, are the words "too large" in shorthand.

On this drawing Linné is represented as a small white spot

* The dark marking which appears on the surface of the Mare about halfway between Linné and Sulpicius Gallus is a stain on the paper, and not part of the original drawing.

† *Selenotopographische Fragmente*, vol. i., plate IX.

without any indication of a crater-like depression. On the southern side a slightly darker shading is faintly indicated, but whether this is intentional or due to some inequality in the paper or other accident it is impossible to say. From its position it cannot be the shadow of a crater ring. As represented in the drawing before us, this object resembles nothing so much as the appearance it offers at the present day with a small telescope and under fairly good conditions of seeing, or on a photograph of this region.

The other drawing, Plate 3, fig. 2, represents on a smaller scale the Mare Serenitatis and the region to the north of it. Unfortunately this is one of the undated pictures, but there is little doubt that it belongs to the same year (1788) as the other, and the state of the shadows shows that it was taken very near the time of full Moon. The only particulars given by Russell with this drawing are the names of three craters near the limb, written in longhand, "Zoroaster, Thales, Endimion" (sic). In this drawing Linné is represented as a conspicuous white spot, with perhaps a slight shading to the south. The absence of any indications of a crater within the white spot in this drawing is not under any circumstances to be wondered at, the illumination being almost vertical at the time. Even Menelaus, Bessel, and Sulpicius Gallus, which in the former drawing appear under an oblique illumination as deep well-marked cavities, are here represented as uniformly white patches. A remarkable feature of this drawing is the relatively large size of Linné. It is here represented as more conspicuous than Bessel, which exceeds it in size in the previous drawing, and about equal to Sulpicius Gallus A. In view, however, of the fact that the first of Russell's drawings was intended chiefly to exhibit the appearance of Bessel, and not Linné, it would be unsafe to attach much weight to the apparent rise in conspicuousness of the latter marking as the phase increases. Such a deduction would be opposed to the result of Professor W. H. Pickering's observations, viz. that it rapidly diminishes in size under the direct solar rays, attaining its minimum diameter about twenty-four hours after lunar midday.

It is not my intention at present to enter upon the general question as to the possibility of a change having occurred in Linné. The principal facts bearing on this point have been recently discussed by Mr. Saunder in his presidential address to the British Astronomical Association, and by Professor W. H. Pickering in the *Annals of Harvard College Observatory*, vol. xxxii., part ii., chap. vii. It is not to be supposed that the observations of such experienced selenographers as Lohrmann, Mädler, and Schmidt (who frequently studied this formation and described it as having been previous to 1843 a large and deep crater, which disappeared in the year 1866) * can be lightly set aside. At the same time the evidence afforded by these drawings of Russell, now published for the first time, corroborates

* See *Report of British Association*, 1867.

in a remarkable manner Schröter's isolated observation, and renders it probable that in the year 1788 this object presented much the same appearance as at the present day when viewed with a low power eyepiece attached to a small telescope.

I would merely venture to point out that the evidence of Grimaldi's two maps, as published by Riccioli in his *Almagestum Novum*, which is sometimes appealed to as proof of the existence of a large crater at this spot about the middle of the seventeenth century, appears to me to be of very little value indeed. In one of these two maps Linné is represented as a white spot, while Bessel and Sulpicius Gallus appear as craters with shaded interiors. In the other map, which seems to be the more carefully finished production, Linné is also provided with a shaded crater-like interior, in every way comparable with that of Sulpicius Gallus. The force of this evidence in favour of the crater-like form of Linné in 1651 is, however, very much weakened, if not entirely dissipated, by the fact that the peaks of the Apennine range, which in the former map are represented as white spots similar to but smaller than Linné, appear in the latter as a row of small carefully shaded craterlets.

Studies of Mersenius and Copernicus—both of which have been suspected of change—are also included amongst Russell's drawings.

Transformation of Hansen's Tables. By P. H. Cowell, M.A.

Newcomb (*Astron. Papers, Amer. Eph.* vol. i.) has transformed Hansen's theory into longitude and latitude in order to compare it with Delaunay. In this paper I give the transformation of Hansen's tables in order to compare them with the observations. Newcomb's paper has, of course, been of great assistance. It will be seen that my method is to vary Newcomb's results, and thus avoid performing the long calculations that would have been necessary in the absence of his paper.

Hansen's longitude is found from the formula

$$\mathrm{L} = f + \Pi + \mathrm{R} + \mathrm{R}'$$

where f, R, R′ are functions of e, I, $n\delta z$, s, and Hansen's latitude β is found from the formula

$$\sin\beta = \sin \mathrm{I} \sin(f + \omega) + s$$

e and I are arbitrary constants to which Hansen assigns one value in the theory and a different value in the tables. $n\delta z$ and s are the symbols under which Hansen calculates the perturbations. As regards these quantities Hansen gives slightly different coefficients in his tables to those that he subsequently

gives in his theory. Moreover in his tables he omits certain terms as not worth tabulating, although in the opening pages of his tables he gives the coefficients that he has calculated, and these coefficients in some cases amount to upwards of a tenth of a second. He has also introduced other modifications such as multiplying all terms in $n\delta z$ by one factor and all parallactic terms by another factor. Hence it must be clearly understood that throughout this paper "Hansen's tables" refers to what Hansen has tabulated, and not to the preliminary expressions.

The variation between Hansen's tables and Hansen's theory will be denoted by the operator Δ.

Hence
$$\Delta e = +0{\cdot}00000728$$
$$\Delta \mathrm{I} = -8''{\cdot}04$$

Also $\Delta n\delta z$, Δs denote respectively the variations of $n\delta z$ and s. Newcomb has considered the parts of $n\delta z$ and s that arise from the action of the Sun only. Hence $\Delta n\delta z$ and Δs consist of two distinct parts, one part arising from the varied coefficients of arguments due to the action of the Sun, and the other part comprising the planetary and figure of Earth and figure of Moon terms in so far as these terms depend on fresh arguments.

These variations will here be considered separately.

I. *The Variation of the Solar Terms.*

Following Newcomb's notation

$$f = (e,\, g)_0 + (e\, g)_1\, n\delta z + (e\, g)_2\, (n\delta z)^2 + \ldots$$
$$\mathrm{R} = -\tfrac{1}{2}\tan^2 \tfrac{1}{2}\mathrm{I} \sin 2\,(f+\omega) + \tfrac{1}{2}\tan^4 \tfrac{1}{2}\mathrm{I} \sin 4\,(f+\omega) - \ldots$$
$$= \mathrm{R}_1 + \mathrm{R}_2 \ldots$$
$$\mathrm{R}_1 = \mathrm{R}_{10} + \mathrm{R}_{11}\, n\delta z + \mathrm{R}_{12}\, (n\delta z)^2 + \ldots$$
$$\mathrm{R}_2 = \mathrm{R}_{2\cdot 0} + \mathrm{R}_{2\cdot 1}\, n\delta z + \mathrm{R}_{2\cdot 2}\, (n\delta z)^2 + \ldots$$

$\mathrm{R}' = s\mathrm{S}$ + three periodic terms that require no transformation + nutation that is not considered in this paper; and

$$s = \mathrm{S}_0 + \mathrm{S}_1 n\delta z + \mathrm{S}_2\, (n\delta z)^2$$

Hence
$$\Delta \mathrm{L} = \Delta(e,\, g)_0 + \Delta \mathrm{R}_{1\cdot 0} + \mathrm{A}\Delta(n\delta z) + \mathrm{S}_0 \Delta s$$

where $\mathrm{A} = (e,\, g)_1 + \mathrm{R}_{11} + s\mathrm{S}_1 + 2(e\, g)_2\, n\delta z + \ldots$

A may be reduced to its first term

$$(e,\, g)_1 = 1 + 0{\cdot}1098 \cos g + 0{\cdot}0075 \cos 2g$$

I also copy from Newcomb's paper

$$S_0 = +0''\cdot005 \cos\omega - 0''\cdot090 \cos(g+\omega) - 0''\cdot005 \cos(2g+\omega)$$

Then

$$\Delta(e, g)_0 = +3''\cdot000 \sin g + 0''\cdot206 \sin 2g + 0''\cdot015 \sin 3g$$
$$\Delta R_{1\cdot0} = -0''\cdot034 \sin(g+2\omega) + 0''\cdot359 \sin(2g+2\omega) + 0''\cdot033 \sin(3g+2\omega)$$

$\Delta(n\delta z)$ and Δs are given in the third columns of the two tables in this paper. The terms in these columns enclosed in brackets are those that do not occur in Hansen's tables; the coefficients are therefore those of the theory with the signs changed. Hence, to form the solar part of ΔL, it is only necessary to perform the multiplications and additions indicated by the formula. Adding to L, as given in the last column of Newcomb's second table, $L+\Delta L$, the solar part of the longitude of Hansen's tables is obtained. The result is set down in the second column of the first table.

The following intermediate step of the calculations is given here.

$$S_0 \cdot \Delta s = -0''\cdot288 \sin(2g+2\omega) + 0''\cdot024 \sin(g+2\omega) - 0''\cdot022 \sin(3g+2\omega) + 0''\cdot018 \sin g$$

Lastly it must be noted that certain coefficients vary with the time. Hence the value of $L+\Delta L$ given in Table I. must be supplemented by the terms

$$-0''\cdot182\, T \sin(-g-g') - 1''\cdot637\, T \sin(-g') - 0''\cdot278\, T \sin(g-g')$$
$$+0''\cdot069\, T \sin(g-g'+2\omega-2\omega') + 0''\cdot057 \sin(2g-g'+2\omega-2\omega')$$
$$-0''\cdot493\, T \sin(g-3g'+2\omega-2\omega')$$
$$-0''\cdot388\, T \sin(2g-3g'+2\omega-2\omega')$$

where T is reckoned in centuries from 1800·0.

For the latitude

$$\Delta\beta = (1+\tfrac{1}{2}\sin^2\beta)\,\Delta \sin\beta$$
$$= (1+\tfrac{1}{2}\sin^2\beta)\{\sin I \sin(f+\omega)\frac{\Delta \sin I}{\sin I} + \sin I \cos(f+\omega)\Delta f + \Delta s\}$$

Of the expression within brackets the first term is obtained from the second column of Newcomb's Table IV. by multiplying by the factor $\frac{\Delta \sin I}{\sin I}$, or $-0\cdot0004328$; its value is set down in

the fourth column of the second table in this paper; in the second term

$$\cos(f+\omega) = \cos(g+\omega) - 0''\cdot1098 \sin(g+\omega)\sin g$$

$$\Delta f = \Delta(e,g)_0 + (1+0\cdot1098\cos g)\,\Delta n\delta z + n\delta z \cos g \,.\, 2\Delta e$$

$$\sin \mathrm{I}\,\Delta(e\,g)_0 = +0''\cdot269 \sin g + 0''\cdot018 \sin 2g$$

$$\begin{aligned}\sin \mathrm{I}\,\Delta n\delta z = {} & +0''\cdot014 \sin(-g') + 0''\cdot043 \sin(-g-2g') \\ & +0''\cdot064 \sin(g-2g'+2\omega-2\omega') \\ & +0''\cdot031 \sin(2g-2g'+2\omega-2\omega') \\ & -0''\cdot013 \sin(-4g'+2\omega-2\omega') + 0''\cdot014 \sin 2\omega \\ & +0''\cdot027 \sin(g+2\omega) \\ & +0''\cdot025 \sin(2g'+2\omega') + 0''\cdot024 \sin(\omega-\omega') \\ & +0''\cdot049 \sin(g+\omega-\omega') \\ & -0''\cdot064 \sin(-g'+\omega-\omega') - 0''\cdot395 \sin(g-g'+\omega-\omega')\end{aligned}$$

and $\sin \mathrm{I}\, n\delta z \cos g \,.\, 2\Delta e$ is less than $0''\cdot01$.

The second term of $\cos(f+\omega)$ is only sensible when taken in connection with the terms

$$+0''\cdot269 \sin g - 0''\cdot395 \sin(g-g'+\omega-\omega')$$

and this part of the product is

$$\begin{aligned}& +0''\cdot007 \sin(-g+\omega) - 0''\cdot015 \sin(g+\omega) + 0''\cdot007 \sin(3g+\omega) \\ & +0''\cdot011 \sin(g-g'+2\omega-\omega') + 0''\cdot011 \sin(g-g'-\omega') \\ & \quad -0''\cdot011 \sin(-g-g'-\omega') - 0''\cdot011 \sin(3g-g'+2\omega-\omega')\end{aligned}$$

The complete value of the term is given in the fifth column of the second table, the excess of what is there set down over the expression just written representing $\cos(g+\omega) \,.\, \sin \mathrm{I}\,\Delta f$.

Δs is given in the third column

$$\tfrac{1}{2}\sin^2\beta \,.\, \Delta \sin\beta = -0''\cdot005 \sin(g+\omega) + 0''\cdot002 \sin(3g+3\omega)$$

and $\beta+\Delta\beta$ is given in the second column of the table, β being taken from Newcomb's fourth table. In both tables only those terms are given which have a coefficient as large as $0''\cdot1$, and the results set down must only be considered as accurate to $0''\cdot02$.

II. *The Planetary and other Terms.*

The planetary terms of $\Delta \mathrm{L} = \mathrm{AP} + \mathrm{S_0 Q}$, where P, Q are the planetary terms of $\Delta\,(n\delta z)$ and Δs respectively.

$$\mathrm{Q} = +8''\cdot764 \sin(f+\Pi+169^\circ\,51')$$

The value of P can be taken from Hansen's tables.

$$\mathrm{A} = 1+0\cdot1098 \cos g + 0\cdot0075 \cos 2g$$

$$\mathrm{S_0} = +0\cdot005 \cos\omega - 0\cdot090 \cos(g+\omega) - 0\cdot005 \cos(2g+\omega)$$

as before.

To this must be added

$$0''\cdot022\cos(g-2g'+2\omega-2\omega')+0''\cdot021\cos(2g-2g'+2\omega-2\omega')$$

multiplied by certain terms of P; for Hansen directs that the sum of the terms in question is to be added to the argument of the evection and to the mean elongation.

The multiplications, so far as P is concerned, need not be performed, for P only contains the following short-period terms:

$$0''\cdot256\sin\Theta\cos g$$

for which A may be taken as unity.

For the long-period terms the product by the term $0''\cdot1098\cos g$ in A is not required for this reason. The observations will be divided into several periods of analysis, and in each period it will be assumed that an error $a\cos g$ runs through the observations. a will be found for each period and compared with $0''\cdot1098$ P.

$$S_0Q=-0''\cdot394\{\sin(2f+\Pi+\omega+169^\circ\,51')+\sin(-\Theta+169^\circ\,51')\}$$

or with sufficient accuracy

$$-0''\cdot394\{\sin(2g+\Pi+\omega+169^\circ\,51')+\sin(-\Theta+169^\circ\,51')\}$$
$$-0''\cdot088\cos(2g+\Pi+169^\circ\,51')\sin g\,.$$

The planetary terms of $\Delta\beta$ are

$$(1+\tfrac{1}{2}\sin^2\beta)\{\sin I\cos(f+\omega)\,P+Q-1''\cdot00\}$$

the last term being a term that Hansen adds to all latitudes on account of the figure of the Moon.

Here also, and for a similar reason, it is not necessary to perform the multiplications.

TABLE I.

Terms in Hansen's Tabular Longitude transformed and in $\Delta(n\delta z)$.

Arg. g	Arg. g'		L+ΔL.	$\Delta(n\delta z)$.	Arg. g	Arg. g'		L+ΔL.	$\Delta(n\delta z)$.
1	0	+	22640″168	...	2	−1	+	9″719	...
2	0	+	769·064	...	3	−1	+	0·670	...
3	0	+	36·127	...	−1	−2	+	+0·709	−0·475
4	0	+	1·932	...	0	−2	+	7·493	−0·014
5	0	+	0·113	...	1	−2	+	2·568	−0·025
−3	−1	+	0·551	...	2	−2	+	0·192	...
−2	−1	+	7·666	...	$2\omega-2\omega'$				
−1	−1	+	109·953	+0·037	0	0	−	0·266	−0·036
0	−1	+	670·006	+0·154	1	0	−	2·488	+0·047
1	−1	+	148·032	+0·024	2	0	−	0·194	...

Arg. g	g'		L+ΔL.	Δ($n\delta z$).
−1	−1	+	0·124	−0·053
0	−1	+	2·556	+0·035
1	−1	−	28·595	−0·036
2	−1	−	24·452	...
3	−1	−	2·926	...
4	−1	−	0·292	...
−2	−2	+	0·949	...
−1	−2	+	13·200	+0·011
0	−2	+	211·704	+0·010
1	−2	+	4586·690	+0·717
2	−2	+	2370·137	+0·352
3	−2	+	191·940	...
4	−2	+	14·374	...
5	−2	+	1·060	...
−1	−3	+	0·453	−0·022
0	−3	+	8·693	+0·033
1	−3	+	206·492	+0·060
2	−3	+	165·556	+0·039
3	−3	+	14·597	...
4	−3	+	1·182	...
0	−4	+	0·134	−0·146
1	−4	+	7·419	−0·013
$2\omega-2\omega'$				
2	−4	+	8·125	...
3	−4	+	0·758	...
1	−5	+	0·257	...
2	−5	+	0·344	...
2ω				
2	1	+	0·416	...
3	1	+	0·265	...
0	0	+	1·253	+0·153
1	0	−	39·281	+0·304
2	0	−	411·598	−0·012
3	0	−	45·080	...
4	0	−	3·997	...
5	0	−	0·329	...
3	−1	−	0·304	..
$2\omega'$				
−1	3	+	0·339	−0·063

Arg. g	g'		L+ΔL.	Δ($n\delta z$).
0	3	−	2·153	...
−2	2	+	0·425	...
−1	2	+	6·435	+0·057
0	2	−	54·980	+0·282
1	2	−	0·139	+0·021
2	2	+	0·561	...
0	1	+	1·499	−0·055
$\omega-\omega'$				
−1	0	+	0·384	...
0	0	+	1·582	+0·266
1	0	+	18·167	+0·551
2	0	+	1·276	+0·011
−2	−1	−	0·118	...
−1	−1	−	1·804	−0·020
0	−1	−	19·150	−0·711
1	−1	−	126·459	−4·388
2	−1	−	8·543	−0·053
3	−1	−	0·588	...
0	−2	−	0·153	+0·014
1	−2	−	0·594	−0·010
2	−2	−	0·127	...
$3\omega-3\omega'$				
2	−2	+	0·270	...
3	−2	+	0·150	...
1	−3	−	1·247	−0·063
2	−3	−	3·224	−0·081
3	−3	+	0·422	+0·027
2	−4	−	0·226	...
$\omega+\omega'$				
0	1	+	0·112	+0·037
1	1	+	0·571	+0·030
$3\omega-\omega'$				
3	−1	+	0·235	(−0·010)
$\omega-3\omega'$				
1	−3	−	0·300	+0·018
$4\omega-4\omega'$				
2	−3	−	0·356	...
3	−3	−	0·640	...
4	−3	−	0·293	...

Arg. g	Arg. g'		$L+\Delta L$.	$\Delta(n\delta z)$.	Arg. g	Arg. g'		$L+\Delta L$.	$\Delta(n\delta z)$.
1	−4	+	1·177	...	4	−2	−	5·753	(−0·010)
2	−4	+	30·778	...	5	−2	−	0·991	...
3	−4	+	38·426	...	6	−2	−	0·124	...
4	−4	+	13·900	...	3	−3	−	0·389	(+0·041)
5	−4	+	1·981	...	4	−3	−	0·384	...
6	−4	+	0·173	(−0·048)	$2\omega-4\omega'$				
2	−5	+	2·746	...	1	−4	+	0·202	−0·021
3	−5	+	4·402	...	$6\omega-6\omega'$				
4	−5	+	1·886	...	3	−6	+	0·292	...
5	−5	+	0·286	...	4	−6	+	0·572	...
2	−6	+	0·156	...	5	−6	+	0·395	...
3	−6	+	0·313	...	$6\omega-4\omega'$				
4	−6	+	0·153	...	4	−4	−	0·150	(+0·016)
$4\omega-2\omega'$					5	−4	−	0·203	...
2	−2	−	0·534	...	4ω				
3	−2	−	9·370	...	4	0	+	0·422	...

Table II.

Terms in Hansen's Tabular Latitude transformed and in Δs, $\sin(f+\omega)\,\Delta \sin I$, $\sin I \cos(f+\omega)\,\Delta f$.

Arg. g	Arg. g'		$\beta+\Delta\beta$.	Δs.	$\sin(f+\omega)\,\Delta \sin I$.	$\sin I \cos(f+\omega)\,\Delta f$.
$-\omega$						
−1	1	−	0″·322	−0″·017	...	...
0	1	−	5·665	...	+0·002	...
1	1	−	6·484	...	+ ·013	...
2	1	−	5·333	...	+ ·003	...
3	1	−	0·640	...	...	...
−2	0	−	1·585	...	...	...
−1	0	−	31·759	...	+ ·003	+0·007
0	0	−	999·533	− ·169	+ ·442	− ·120
1	0	+	18461·652	+6·405	−7·981	− ·015
2	0	+	1010·006	+ ·136	− ·437	+ ·134
3	0	+	61·891	+ ·010	− ·027	+ ·007
4	0	+	3·977	...	− ·002	...
5	0	+	0·263	...	...	...
−1	−1	+	0·312	...	...	...
0	−1	+	5·102	− ·022	− ·001	...
1	−1	+	4·867	...	− ·013	...
2	−1	+	6·759	...	− ·004	...

Arg. g	Arg. g'		$\beta+\Delta\beta$.	Δs.	$\frac{\sin(f+\omega)}{\Delta \sin I}$.	$\frac{\sin I}{\cos(f+\omega)}\Delta f$.
			"	"	"	"
3	−1	+	0·798	...	...	...
2	−2	+	0·142	(+ ·026)	...	...
$\omega-2\omega'$						
0	−1	−	0·797	...	...	...
1	−1	−	12·140	...	...	...
2	−1	−	0·832	...	...	...
−3	−2	+	0·134	...	...	...
−2	−2	+	1·518	...	− ·001	...
−1	−2	+	15·570	+ ·019	− ·009	...
0	−2	+	166·581	+ ·037	− ·091	+ ·032
1	−2	+	623·708	+ ·045	− ·043	+ ·004
2	−2	+	33·368	...	− ·001	...
3	−2	+	2·146	...	...	...
4	−2	+	0·145	...	...	...
−1	−3	+	0·655	...	...	...
0	−3	+	7·471	...	− ·004	...
1	−3	+	29·733	...	− ·003	...
2	−3	+	1·776	...	...	...
3	−3	+	0·121	...	...	...
0	−4	+	0·276	...	...	...
1	−4	+	1·095	...	...	...
$\omega+2\omega'$						
0	2	+	0·286	...	...	...
1	2	−	2·151	+ ·029	+ ·002	+ ·012
2	2	−	0·326	...	...	...
3ω						
1	0	+	0·108	...	...	...
2	0	−	2·790	...	+ ·002	+ ·014
3	0	−	6·301	...	...	...
4	0	−	1·021	...	...	...
5	0	−	0·119	...	...	...
$3\omega-2\omega'$						
2	0	−	0·116	...	...	...
2	−1	−	1·320	...	+ ·001	...
3	−1	−	1·276	...	+ ·001	...
4	−1	−	0·257	(− ·017)	...	...
0	−2	+	0·269	...	...	...

Arg. g	Arg. g'		$\beta+\Delta\beta$.	Δs.	$\sin(f+\omega)\,\Delta\sin I$.	$\sin I\cos(f+\omega)\,\Delta f$.
			″	″	″	″
1	−2	−	1″622	...	+ ·001	...
2	−2	+	199·455	+ ·027	− ·087	+ ·032
3	−2	+	117·243	+ ·020	− ·051	+ ·016
4	−2	+	15·124	(+ ·015)	− ·006	...
5	−2	+	1·518	...	− ·001	...
6	−2	+	0·140	...	...	...
2	−3	+	8·908	...	− ·004	...
3	−3	+	7·993	...	− ·004	...
4	−3	+	1·162	(+ ·022)	...	...
5	−3	+	0·127	...	...	...
2	−4	+	0·320	...	...	...
3	−4	+	0·390	...	...	...
$3\omega-4\omega'$						
2	−3	−	0·153	...	...	...
3	−3	−	0·103	...	...	...
1	−4	+	0·642	+ ·017	...	...
2	−4	+	6·576	...	...	...
3	−4	+	3·679	...	...	...
4	−4	+	0·466	...	...	...
2	−5	+	0·517	...	...	...
3	−5	+	0·410	− ·012	...	...
$-\omega'$						
0	0	+	0·802	(− ·015)	...	+ ·024
−2	−1	−	0·110	...	...	...
−1	−1	−	0·467	+ ·010	...	− ·054
0	−1	−	4·906	− ·020	...	− ·198
1	−1	−	0·606	− ·022	...	...
$2\omega-\omega'$						
2	0	+	0·812	...	...	+ ·024
3	0	+	0·101	...	...	...
1	−1	+	0·122	+ ·029	...	− ·032
2	−1	−	5·412	+ ·037	...	− ·198
3	−1	−	0·672	...	...	− ·022
$2\omega-3\omega'$						
1	−3	−	0·295	...	...	...
2	−3	−	0·351	...	...	...
$5\omega-4\omega'$						

Arg. $g \cdot g'$		$\beta+\Delta\beta$.	Δs.	$\frac{\sin(f+\omega)}{\Delta \sin I}$.	$\frac{\sin I}{\cos(f+\omega)\,\Delta f}$.
		′	″	″	″
3 −4	+	2·419	...	...	...
4 −4	+	3·004	...	...	...
5 −4	+	1·193	...	...	...
6 −4	+	0·214	...	...	...
3 −5	+	0·218	...	...	...
4 −5	+	0·346	...	...	...
5 −5	+	0·161	...	...	...
$4\omega-3\omega'$					
3 −3	−	0·208	...	...	...
$5\omega-2\omega'$					
4 −2	−	0·246	...	...	...
5 −2	−	0·145	...	...	...

A Spectrographic Study of β Lyræ. By Rev. Walter Sidgreaves, S.J., Stonyhurst College Observatory.

Introduction.—The more the spectrum of *β Lyræ* is studied, the more evident it becomes that its photographs cannot be too greatly multiplied ; and one of the objects of this paper is to show that a suitable telespectrograph would be well employed if devoted solely to this purpose until a series of plates had been collected containing many for each day of the light-period.

The attempt made here during the last two years has been greatly frustrated by the extraordinarily cloudy state of the nights. And the result, so far, is disappointing, inasmuch as many more photographs are wanted, in order to support or undermine the constructive explanations suggested during a rather lengthy study of the plates already collected.

All the photographs have been taken with the small prismatic camera of 4 inches aperture described in the December number of the *Monthly Notices*, 1901, in the introductory part of the paper on *Nova Persei*. They cover the spectrum from Hβ to the head of the hydrogen series, and on one of the stronger photographs the lines can be counted up to Hπ. But Hβ and the lines beyond Hη are too weak for any trustworthy comments, and the enlargements have been cut down accordingly.

Professor Vogel, discussing the Potsdam photographs of

1892–3,* has noted the importance of correct exposure in all scientific applications of photography. Defect of exposure has been a source of loss to the Stonyhurst series of spectrograms of 1902–3. The slitless spectrograph employed here has its own merits, but for successful exposures the slit has the advantage in this, that the time of exposure can be prolonged at will, to suit the state of the sky and the magnitude of the star. With a slitless spectrograph exposure can be quoted only as a velocity ; and this is limited in slowness by a small oscillation of the star's image, caused by inequalities of the clock's driving-screw. The rate of trail must not be less than the amplitude of this oscillation in the time of the oscillation ; otherwise the stellar image backs on the plate periodically and blurs the spectral lines.

Another source of loss to the series of spectrograms has been the difficulty often experienced in setting the spectrum on the plate in its true position of focus, owing to the considerable slope of the plate required by the spectral image as given by a visual objective. This presents no difficulty when the sky is really clear and the spectral colours can be distinguished ; but real clearness has been exceptional in the last two years.

The criterion adopted for the selection of the plates has been a readable enlargement, and this has thrown out of service about 40 per cent. of the total number of plates.

These enlargements have been found of great service in studying the changes of the several lines. They are not of the same worth as the original negatives, but after an examination of each of the negatives in conjunction with its enlargement, the details of the negative can be recognised on the positive, and then comparisons are easily made on the card of mounted enlargements. But it should be understood that all remarks upon the lines in the course of this discussion of the spectrum are made on the original negatives, and these remarks refer only to the selected negatives ; but the rejected plates are not all worthless, and a list of these is appended at the end of the paper, with remarks concerning their positions on the spectrographic chart or card of enlargements.

The order of the spectrograms on the chart is that of their time-intervals in days and hours from the previous principal minimum of the light period. These intervals are entered in the margin, and will be referred to as *periodic dates*, indicated generally by the letter D written after the date ; and the civil date can be taken from the opposite margin, in which the date of the previous principal minimum is entered.

Mounted in this order on a long strip of cardboard the enlargements possess a double value—first, for the study of changes in connexion with the light period ; and second, as a reference chart for subsequent photographs of the spectrum.

* *Über das Spectrum von β Lyræ*, Königl. Preuss. Akad. der Wissenschaften, vi. 1894.

A very cursory examination of the chart is enough to discover a regularity in the sequence of spectral changes so independent of absolute time-intervals that a word of caution seems necessary to protect the reader against erroneous conclusions which may be suggested by this arrangement of dates. It was probably this risk which prompted Professor Vogel's remark at the end of the paper already referred to: "Dass nach den hier gemachten Beobachtungen es nicht zulässig sein dürfte, Beobachtungen, welche Monate lang aus einander liegen, zu verbinden, um die Veränderungen der Linien innerhalb der Lichtperiode darzustellen." An illustration taken from the first plates of the series will serve this purpose. On the periodic dates 0–0 and 0–5 D the line Hζ consists of three parts, a dark centre and two bright sides. On 0–6 D the same line has only one bright side. There is no proof here of so sudden a change as that indicated by one hour difference of periodic time, for the absolute interval is nearly one year. But there is some probability of suddenness represented by the present state of the chart, which shows three components of the line on the two plates of 0–0 and 0–5 D, and only two components on 0–6, 0–16, and 0–17 D; and if we had the confirmation of this change by many photographs of different absolute dates belonging to the same two periodic intervals, the probability of the sudden change would become very great. This confirmation is wanted for many of the inferences suggested in the following comments on this series of spectrograms, which is far from complete. Some of the inferences may be already contradicted by the photographs obtained with other instruments; but discordances of different instruments should not be too easily set up as contradictions. A comparison series of this sort, in which the parts were filled in by instruments of unequal dispersion and light-power, would be for many conclusions entirely misleading.

Before discussing the changes exhibited on the photographs it is necessary to challenge a proposition in Miss Clerke's *Problems in Astrophysics*, p. 344, "that a dark line in the spectrum of one star cannot cut a slice out of a brilliant band proceeding from another." The proposition is an answer to a question of primary importance to the interpretation of the changes in the spectrum of *β Lyræ*; for, if it cannot be gainsaid, the simple explanation of those changes which are periodic by relative motion of two stars must be abandoned, and nothing remains, at least to the writer, to take its place. Changes of pressure seem wholly inadequate, and their periodicity inexplicable. This must be the apology for the following, perhaps lengthy, discussion of the question.

Every dark line in the solar spectrum is dark only by contrast with the adjoining continuous spectrum, and it matters nothing whether the contrast is the result of subtraction or of addition. In the Sun or single star it is by subtraction (absorp-

tion) of a fraction of the light-intensity. In the case of two stars in the same sight-line it is by addition.

Let i and i' be the intensities of the continuous spectrum in the neighbourhood of λ by each of the two stars S and S′; and Δi, $\Delta i'$ the differences producing line λ. Then

the intensity of bright λ by S is $i+\Delta i$

and

the intensity of dark λ by S′ is $i'-\Delta i'$.

In superposition the intensity of the continuous spectrum is

$$i+i'=c$$

and that of λ is

$$i+\Delta i+(i'-\Delta i')=l$$

and λ is bright or dark according as $l \gtrless c$, *i.e.* as $\Delta i \gtrless \Delta i'$.

It seems clear therefore that bright and dark λ can be neutralised; for Δi can $=\Delta i'$. Therefore if the bright line is broad, and the dark line narrow, the contrast can be between neutralisation of the centre and integrated lateral brightness, and the contrast becomes greater when $\Delta i' > \Delta i$. This is the case with Hζ in the spectrum of *β Lyræ* Professor Vogel has the dark component "absolut dunkel"* on the photographic plates, which is expressed by $\Delta i' = i'$. The intensity, then, of λ at the centre of the broad bright line becomes

$i+\Delta i$ between sides of intensity $i+\Delta i+i'$.

It must remain a matter of opinion whether there is here contrast enough for the appearance of a strong dark line crossing the broad bright line. It cannot be black physically, for its intensity is $i+\Delta i$; but can it *look* black by contrast? The writer thinks it can: for the red hydrogen line of the Sun is very brilliant, as seen *off* the solar limb, and this intensity is the least possible light-intensity of the line, which looks very dark *on* the solar surface. But, finally, can a physically bright line, which looks dark by contrast, fail to make its impression on the photographic film, and leave, as on the Potsdam plates, a line of clear glass? This is a matter of exposure. Time of exposure is the photographic analogue of ocular contrast. Vogel insists emphatically on correct exposure, presumably for the general picture; and this may be the exposure which leaves dark Hζ, on the negative clear glass. The slit spectrograph could decide the case of Hζ in *β Lyræ* by timing the exposure needed for a perceptible deposit of silver on the line. If time should fail, the consequence is clear: that the dark line can only be the result of absorption in the one star; and the importance of this determination needs no words.

* *L.c.* p. 120.

Hζ.—The line at the position of the hydrogen line Hζ is well described by Vogel as "bei weitem die auffallendste Linie in dem durch Photographie fixierten Spectrum."* But our photographs suggest the addition that its prominence in the spectrum is chiefly, if not wholly, due to its dark component. The line λ 447 $\mu\mu$ appears for the greater part of the light-period as the brighter line, and it passes through greater changes. But the dark component of Hζ has no competitor: it is unique in the spectrum. On some plates it appears, as described by Vogel, a clear-glass line. It is sharply cut, and it is narrower than any other of the hydrogen lines, except when Hγ and Hδ are nearly obliterated at the chief minimum, and when they are reduced to fine sharp lines soon after the secondary minimum of light.

During the first part of the light-period, from 1–11 to 6–20 D, bright Hζ differs in no appreciable manner from bright Hγ; and during the rest of the period the apparent difference between the two bright lines is attributable to the real difference between their dark lines. In the first part both bright lines are seen only on the red sides of their dark companions. On the five dates 7–4 to 7–19 D the bright component of Hγ is extinct, and that of Hζ is very weak and divided. The narrow dark Hζ cuts out a more or less central part of the broad bright line, while the more diffuse Hγ absorption extinguishes either the whole bright line when concentric, or one of its sides when relatively shifted.

The change from the appearance of a single to that of a double bright line begins on 5–9 D. On this date there is, on the more refrangible side of the dark line, a clear bright fringe, which is also seen on 6–20 D, but less clearly. On 7–4, 7–6, and 7–15 D, Hζ is a very feeble broad bright line, divided, perhaps centrally, by the dark component; but it is too weak for measurement. It is stronger on the plates of 7–17 and 7–19 D, but divided unequally, the red side part being the stronger and broader. And it retains the appearance of a double bright line throughout the rest of the period, including the principal minimum, up to 0·6 D, on which date is found the first return to the condition of a single bright line on the red side. The two bright parts are of nearly equal strength at the chief minimum; the red side part is the brighter on 0·5 D, and on 0–6 D the shorter wave part has disappeared.

This disappearance might be the effect of a relative shift of the dark line over the single broad bright line, or it might be the simple extinction of the more refracted of a pair of bright lines. The width of the remaining bright line, on the red side, measured on the negative, seemed to favour the latter possibility, for it was not wider than before the disappearance of its more refracted companion. And the position of the vanished line was found to be at λ 388·1 $\mu\mu$, the position of the strongest of Cyanogen lines;

* *Loc.* p. 120.

suggesting a periodic extinction and reappearance of this line, as was observed for a while in the spectrum of Nova *Persei*.

On this account very careful measures were made of the widths of the components of Hζ on all the photographic plates of the series, in order to compare them in the two conditions of the whole composite line—viz. first, when the bright line is seen on both sides of the dark line, and secondly, when it is seen on one side only. Measures of this sort cannot be altogether free from errors, attributable to varying conditions of the atmosphere and to relative excellence of the photographs. But these errors may be expected to balance one another in a large number of photographs taken on different nights. The results are presented in abridged form in the following table, taken from the details contained in the complete table at the end of the paper. In both tables the widths are entered in micrometer divisions (centimillimetres), and the periodic intervals in days and hours. In this abridged table the series is divided into four parts, according to the apparent uniformity of the widths.

Table of mean widths of the whole composite line Hζ.

	d	h		Periodic intervals. d	h		d	h	No. of dates.	Mean width.	Greatest width.	Least width.
Chief min.	12	22	...	From 12	12	to	0	5	5	20·8	22	18
First max.	3	2	...	0	6		2	13	11	15·5	17	14
Second min.	6	11	...	2	20		9	14	23	22·0	24	18
Second max.	9	18	...	10	0		12	12	11	16·5	18	15

From this analysis it appears that if the two bright lines, as seen one on each side of the dark line, belong to the same origin, the resulting broad band changes between twenty-one and sixteen units of width. We have therefore to choose between a temporary loss of width in bright Hζ and a temporary extinction of another line, possibly CN. In favour of the latter hypothesis a strong case would have been made out if the greater breadth of the line had been maintained through the last quarter of the period; but it must be observed that during this quarter all the bright lines are failing in strength, and bright Hζ may, on this account, have lost width on the more refractive side, which is weaker than the red side and shades down to the continuous spectrum. Accepting, provisionally, this explanation, we should have for the normal width of the whole line about twenty-one divisions, and this cut down to sixteen only for about two days shortly after the principal minimum. The extinction of a line during these two days could then be accounted for by dissociation at higher temperature, as in the writer's explanation of the same extinction in Nova *Persei*, but occurring, unlike the Nova, near the minimum, instead of at the maximum, of the light curve. There is, however, no contradiction in this difference, the light-changes being by eclipses in *β Lyræ* and by temperature in the

Nova. In both stars the extinction would be by dissociation effected by greater heat; and in *β Lyræ* greater heat is quite possible in conjunction with less light, more light being lost by eclipse than is gained by heat.

Tidal disturbance of a near approach at the principal minimum is perhaps the only assignable cause of the postulated rise of temperature, for its periodic recurrence can only be explained by the regularity of orbital motion; and although this supposition seems to be excluded by the form of the light-curve, which requires uniform velocity of revolution, it will not be out of place to point out in what ways the near approach of the two stars would better agree with the periodic changes of the spectrum. For at present there are, in every hypothesis, forbidding difficulties, which drew Professor Vogel's remark: "Ich halte den Zeitpunkt, eine einigermassen erschopfende Erklärung der sehr complicirten Erscheinung zu geben, noch nicht für gekommen."* And it is only by collating suppositions, however antagonistic, which better account for separate facts, that we may hope to hasten the time of arriving at the truth.

In the first place, the separation of the bright and dark lines so soon after the principal minimum is more in accord with the sharper turn at the periastron of an elliptical orbit than with the rounded transit of a circular path. In the latter motion the component velocity in the sight-line would change too slowly to give the observed displacements so early in the period, for the separation of the lines is well advanced within thirty-six hours after their conjunction at the minimum.

2. The second conjunction of the lines occurs after the secondary minimum of light, between 6–20 and 7–20 D; and this retardation on the conjunction of the two stars could be explained by an elliptic orbit having its major axis inclined a little to the sight-line in the plane of the orbit.

3. There would be an outburst of incandescent hydrogen and helium, &c., shortly after the nearest approach of the stars, giving the strong bright lines, while the higher temperature would kill the CN and its bright line. In the cooling process throughout the rest of the period the CN band would reappear and the other lines would fade. In all respects the changes would be according to the general appearances of the photographic chart.

4. This real increase of intensity is a more satisfactory explanation of the exceeding brightness of the hydrogen and other lines, shortly after the chief minimum, than contrast with a weaker continuous spectrum; for this contrast ought to be the same before and after the minimum, and the lines should be growing into prominence as the minimum is approached, whereas, according to the photographic chart, they are steadily fading all through the last quarter of the period.

* *L.c.* p. 130.

Hγ.—The hydrogen line Hγ has been already described along with Hζ, but some details have yet to be filled in. For some hours before and after the principal minimum the line is single and dark, but very weak. Both components, bright and dark, come into prominence as strong thick lines rather later than those of Hζ. The presence of the bright line is doubtful on 0–17 D of 1903–7–24 on a plate which shows Hζ and other lines very well; but on 0–16 D of 1902–9–17 both components are easily seen on a thinner but clearer photograph. Both components are in fullest strength, for the first time in the series, on 1–11 D. The intensity of the bright line falls off considerably, but there is no certainty of any change, either in the strength of the dark line or in the relative position of the two lines, until after the second minimum. The bright line remains fully on the red side through twenty-five successive periodic dates, including 6–20 D, nine hours after the second minimum. On 7–4 D the transition has begun, and Hγ is a single sharp fine dark line on 7–4, 7–6, 7–15, 7–17, and 7–19 D. The bright line reappears on 8–3 D wholly on the more refractive side of the dark line, where it remains without any apparent change of position until it vanishes some hours before the principal minimum. It is seen on nineteen successive periodic dates up to 12–3 D, when it has only a doubtful existence, and after this date it is quite extinct on all the seven plates covering the principal minimum.

There is therefore, in the behaviour of bright Hγ, fair evidence, first, of a retardation of the spectroscopic upon the eclipsing conjunction of the stars, of about twenty-four hours, the concentric superposition of the bright and dark lines occurring between 6–20 and 8–3 D, and the secondary minimum on 6–11 D; and secondly, of the general decline of bright-line intensity during the course of the light-period.

Hδ and Hε.—Both these lines follow Hγ in all the changes; but their bright components are much weaker, and probably on this account the dark lines recover strength after the secondary minimum better and sooner than that of Hγ.

The thinner dark companion line on the more refractive side of Hε gives to the hydrogen line the appearance of a double absorption line before and after the principal minimum, contracting to a single line, in the middle of the light-period. But the companion line is always well separated from Hε. It is a strong line only between the principal minimum and first maximum. For the rest of the period it remains a very fine line, not visible on all the plates, but becomes clearer towards the end of the period. The apparent separation of the two lines, however, differs very definitely in the two halves of the light-period; and the difference confirms the evidence of Hγ that the spectroscopic conjunction occurs later than the second stellar conjunction. In the first half of the light-period, from 0–0 to 6–20 D the mean separation is 13 divisions of the micrometer head, between extreme readings 12 and 14, from 14 plates; in the second half,

from 7–4 to 12–15 D, the mean separation is 19 divisions between extremes 15 and 22, from 12 plates; and the mean of these two is the measure on the plate of 7–4 D. But this mean may reasonably be taken for the normal separation of the lines when unaffected by displacements. And thus 7–4 D appears as the periodic date of no displacement. Unfortunately, on account of the weakness of the companion line at mid-period, we have only the one measure, on 7–4 D, between dates 5–9 and 9–1 D; but on this plate both lines are thin and sharp.

It must be noted, in addition, that the changes of separation of these two lines cannot be attributed to a Doppler effect of motion. In the first half of the period, when the separation is less, the bright component lies on the red side of the dark line, cutting down its width and thus shifting its apparent centre nearer to the companion line, an effect which is reversed in the second half of the period, when the separation is greater.

Elliptic orbit.—The spectroscopic evidence, therefore, in favour of an elliptical orbit is not small. The superposition of the bright and dark hydrogen lines does not coincide with the secondary light-minimum, but occurs some hours later, between 7–4 and 7–19 D; while at the principal minimum the coincidence is perfect; and these are the conditions of elliptic orbital motion, with the major axis inclined to the line of sight in the plane of the orbit: the angle being formed by rotating the axis in the direction of the orbital motion. Then, according to the excentricity of the ellipse, and the inclination of the axis, we may have the spectral conjunction considerably later than the stellar or eclipse conjunction near apastron, and no measurable difference between them at periastron: as may be seen in the diagram at the end of the table.

Radial velocity.—It is now generally recognised that the relative radial velocity of the components of a binary star cannot be determined by the displacement of lines which are too broad to escape overlapping. The positions of their centres cannot be fixed, and there is the additional uncertainty of the form of the widening, whether equally or unequally on either side of the true position in the spectrum. But we may derive an approximate measure of the relative velocity of the pair in *β Lyræ*, independently of the spectral centres of the lines, through the opposite displacements of bright Hζ before and after the secondary minimum of light. The character of Hζ makes this possible. The dark line has no diffusive edges, and the red side margin of the bright line is never concealed. The centre, therefore, of the dark line may be taken as a fiducial point for the measurement of the relative displacement of the red side edge of the bright line, caused by the opposite velocities of the same star, with reference to the other, at the two epochs of the light curve before and after the secondary minimum. These measures are taken from the table of widths at the end of the paper by adding half the width of the dark line in Column II. to that of the bright

line in Column III., and are entered in column r. An inspection of these measures shows that the greatest elongation of the red side edge of the bright line from the centre of the dark line occurs between 2–20 and 6–20 D, and the least between 10 and 11 D. The mean of the greater measures (r_1) from 18 plates is 16·3 divisions, at epoch 1, and the mean of the smallest measures (r_2) from 4 plates is 7·5 divisions. The difference is the displacement effected by the arithmetical sum of the two opposite velocities of the bright-line star, with reference to the other, at the two epochs; and half this difference is the displacement due to the relative velocity of the pair of stars at either *one* of the two epochs. All the measures were made by the writer, so that personal equation disappears in the differences, and the only systematic error that might vitiate the result is a possible shift of the apparent centre of the dark line by the position and strength of the bright line. Between the first maximum and second minimum, when the measures are greatest, the bright line lies for the most part so completely on the red side of the dark line that it might cut down its width on this side only; and thus the apparent centre of the dark line would be shifted away from the red side edge of the bright line. This would give the readings of r_1 too great, while the smaller readings (r_2) would be unaffected, the bright line being seen well on both sides, and the consequent thinning of the dark line presumably symmetrical. A correction therefore is wanted for the greater measures (r_1), and this may be taken from the measured widths of the dark line entered in the table of widths. The mean width at epoch (1) is 8·0 divisions, and at epoch (2) 6·0. Half this difference, or 1·0, is to be deducted from the mean of the greater measures, and we have for r_1 corrected 15·3 divisions, and for r_2 7·5. The difference 7·8 is the total displacement between the two epochs. The half, or 3·9, is the displacement at either epoch; and the resulting relative velocity of the two stars is 120 kilometre-seconds, which, on the supposition of equal velocities, gives 60 km. secs. for the velocity of either star.

No high degree of probability can be claimed for this measure of the velocity: it needs the confirmation of more photographs; but the method seems worthy of consideration, and might command confidence if the details were provided by an instrument of greater light-power. The result, however, as it stands at present is a remarkable, although perhaps to some extent fortuitous, confirmation of Belopolsky's measure of the apparent orbital velocity of the bright-line star. The Pulkowa measures were made between the bright lines, Hβ, as given by the star and by a Geissler hydrogen-tube; and the resulting orbital velocity in circular motion is 12·01 lieues géographiques,* or 66·7 km.-secs, to compare with the 60 kms. by the Stonyhurst measures.

Group 447–8.—This group of bright and dark lines presents

* *Le spectre de l'étoile variable β Lyræ : Mélanges mathématiques et astronomiques*, t. vii. liv. 3, p. 433.

a very perplexing study. It comes into prominence synchronously with $H\gamma$, early in the period, and dies away with it at the end, so that its cycle of changes is complete in the same light-period. But the changes in the grouping of the components are not easily accounted for on the single supposition of a simple binary star.

There are three principal phases of the grouping : First, *early in the period*, a strong bright line between two dark lines of unequal strength, the weaker one being on the red side of the group ; secondly, *an intermediate phase*, when only a single, narrow, and sharply cut bright line is seen ; and, thirdly, *after the second minimum*, a broad bright band divided by a fine dark line, and terminated on the red side by a stronger absorption.

The intermediate phase appears to correspond with that of the hydrogen lines, differing only by showing the single bright line instead of the single dark line ; but it occurs many hours earlier, halfway between the first maximum and second minimum, as seen on 4–16 to 4–20 D. On 6–20 D, nine hours after the second minimum, the bright line is already broad, divided, and terminated on the red side by a dark line as in phase 3. The phase is fully developed on 7–15 D ; and the group retains this form repeated on twenty-five successive periodic dates to the end of the light-period, except that the bright parts fade away towards the end, and are quite extinct on the five last plates, from 12–3 to 12–15 D ; when also the dark lines appear in equal strength, but much weaker than at the beginning of the period.

The two dark lines occupy the same positions with reference to dark $H\delta$ probably all through the period ; but they are not visible on the plates of the intermediate phase, and are too ill-defined for the micrometer on six of the plates after the second minimum. The measures on nine plates before the second minimum, and on seventeen plates after it, assign their positions in the spectrum at those of helium and magnesium, 447·1 and 448·2, the readings varying irregularly between 446·9–447·3 for the former, and between 448·1–448·5 for the latter.

The appearance of the group in the third phase, after the second minimum, is quite that of a pair of bright lines (helium and magnesium), each with its dark companion on the red side, in keeping with the hydrogen displacements. But these displacements should be reversed after the principal minimum. There should be a bright companion on the red side of the dark magnesium line ; and there is no sign of this, although the two bright lines in phase 3 are of equal strength. Also the intermediate phase has no explanation on this supposition. And, lastly, the relative displacements of both pairs of lines would be in opposition to those of hydrogen on the same plate of 6–20 D.

A better agreement between the phases of the group might be found on the supposition of a single broad bright line, which after the principal minimum, in phase 1, covers the region between the red side limits of the two dark lines, paling the

magnesium and leaving the helium in fullest strength. The width of the whole bright line, as seen in phase 3, is rather more than enough for this ; and the changes in the relative intensities of the two dark lines favours the supposition ; for as the relative shift proceeds, the bright line uncovers the magnesium line and covers the helium line, until on 5–9 D and following periodic dates the magnesium is the stronger dark line. Also the intermediate phase of a single bright line would appear when the broad bright line covered both dark lines nearly equally ; and the periodic time of this occurrence, being considerably earlier than that of the mean position of the oscillating line, would be in accord with the date of this phase in the period. And, finally, the mean position of the line would show a darker magnesium line and a broader bright line divided by the helium absorption, as seen on the plates of 7–4 and 7–6 D : a result of considerable weight in favour of this supposition, seeing that it brings the periodic time of the mean position of these lines into agreement with that of the hydrogen lines, and accounts for the apparent contradiction already noticed on plate 6–20 D of the hydrogen bright lines and those of this group in seemingly opposite displacements.

Conclusion.—There are other manifestations on some of the photographs which indicate, on the principle of Doppler displacements, temporary disturbances similar to the solar eruptions which distort the hydrogen lines. A line is bent or curved while the neighbouring lines are quite straight. The effect on a slit spectrograph would be a wider line, but the trailed line shows the progressive change of the line. It may, however, be delusive, a mere photographic freak ; and the only demonstration possible would be the same effect by two instruments running at the same time—a coincidence not to be looked for. But the possibility of a real stellar cause of the distortion is a possibility too probable in itself to be neglected. And for this reason, returning to the opening paragraph of this paper, it may be with some confidence asserted that the suitable telespectroscope for *β Lyræ* must be the slitless prismatic camera of as short focal length as may be consistent with sufficient light-gathering aperture. The 4-inch prism employed here is only just enough, and fails at the important epochs of least magnitudes, except in exceptionally clear atmospheric conditions ; and the disappointments are too frequent to hope for continued perseverance in the work. The ideal instrument would be a 5-inch prism and objective corrected for the photographic spectrum ; the prism mounted for first surface refraction only, and the second surface figured as part of the corrected objective ; the refractive angle of the prism and the focal length of the objective being adjusted to give length of spectrum from Hβ to Hη, about 2 centimetres. With an instrument of this description the bright lines would not suffer in contrast by over-magnification; disappointments would be exceptional, and the work could be carried on with the regularity

of Sun-spot photographs, supplying the material for a fruitful study of the variations of this star, which, as Vogel has it, "zält in spectralanalytischer Beziehung zu den interessantesten Objecten des nördlichen Himmels," and without which we cannot hope to unravel "the endless complications of the problem star." *

Notes on the following table of Hζ *components.*

D = Periodic date = interval from previous principal minimum, in days and hours.
M = Civil date (year, month, day) of previous principal minimum.
I., II., III stand for the components of Hζ in order of increasing wave-length; II. being the dark line.
W = Width in micrometer divisions.
S = Sum of the widths, or total width of composite line.
r = Length from apparent centre of dark line II to red side edge of bright line III.
a. Another photograph on the same night gave all the measures greater. The photograph was very weak, by a trail much too quick.
b. These two plates have been re-examined many times, on account of the seeming greater total width of Hζ on the positive enlargements. The measures entered always appeared to be the most probable readings.
c. The plate of 4–6 D is omitted from the table on account of a scratch on the film which passes through the important line III., and makes impossible a true pointing of the margin. But it is most probable that the width of III. is exceptionally small.
d. Probable fine bright edge.
e. Probably broad; not measurable.
f. Doubtful existence.

Hζ Components.

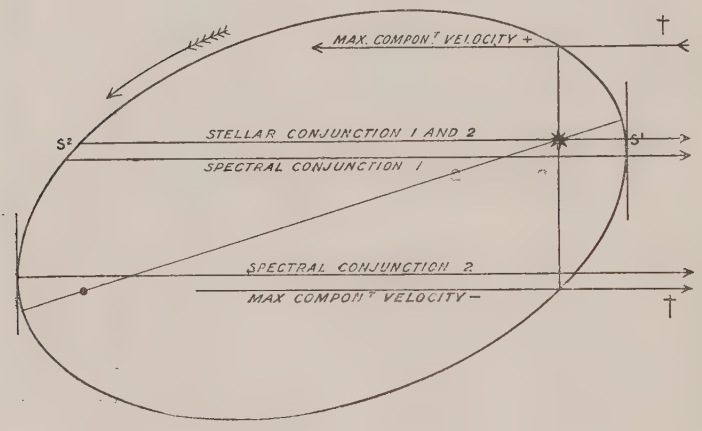
	D. d	D. h	M. y	M. m	M. d	I. w.	II. w.	III. w.	S w.	r
	12	12	1902	8	22	8	5	8	21	10.5
	12	15	2	9	4	5	7	6	18	9·5
Chief min.	12	22	2	5	23	8	6	7	21	10·0
	0	0	3	9	13	7	7	8	22	11·5
	0	5	1	8	25	9	6	7	22	10·0
	0	6	2	7	1	...	8	6	14	10
	0	16	2	9	17	...	8	7	15	11
	0	17	3	7	24	...	8	8	16	12
a	1	3	3	10	22	...	8	7	15	11
b	1	11	2	8	9	...	7	9	16	12·5
b	1	14	2	9	4	...	9	8	17	12·5
	2	0	3	9	13	...	8	7	15	11
	2	12	1	10	29	...	8	8	16	12
	2	12	2	8	22	...	8	7	15	11
		13	2	8	22	...	8	8	16	12
	2	20	1	6	22	...	8	12	20	16
	2	22	3	8	19	...	9	11	20	15·5

* A. M. Clerke, *Problems in Astrophysics*, β *Lyræ*.

D. d h	M. y m d	I. w.	II. w.	III. w.	*s* w.	*r*
Max. 1 (3 2)		...	...	...	...	...
3 12	1 10 29	...	8	13	21	17
3 12	2 8 22	...	8	10	18	14
3 14	2 9 4	*d*	8	11	19	15
3 18	1 6 9	3	7	15	25	18·5
3 19	2 9 30	*d*	7	13	20	16·5
3 21	3 8 19	*d*	9	12	21	16·5
c 4 12	1 10 29	*d*	8	12	20	16
4 15	1 11 24	*d*	7	15	22	18·5
4 16	1 5 27	*d*	8	14	22	18
4 20	1 6 22	*d*	10	12	22	17
4 21	2 10 25	3	8	11	22	15
4 21	3 8 31	*d*	9	11	20	15·5
4 23	3 9 13	*d*	9	12	21	16·5
5 9	2 7 27	3	6	11	20	14
5 23	3 8 31	3	8	13	24	17
Min. 2 (6 11)		...	...	...	...	...
6 20	2 10 25	2	9	12	23	16·5
7 4	1 8 12	10	5	9	24	11·5
7 6	1 9 7	*e*	4	9	...	11
7 15	2 9 17	*e*	6	8	...	11
7 17	2 10 13	8	8	8	24	12
7 19	2 10 13	7	5	9	21	12·5
8 3	1 8 12	6	6	*e*	...	...
9 1	1 7 17	6	7	*f*	...	...
9 14	1 5 14	6	5	11	22	13·5
9 14	3 6 8	5	6	8	19	12
Max. 2 (9 18)		...	...	...	...	..
10 0	2 5 23	5	7	4	16	7·5
10 3	1 8 12	5	7	4	16	7·5
10 21	3 8 31	5	5	5	15	7·5
10 23	3 8 31	5	5	5	15	7·5
11 10	2 8 9	5	6	5	16	8·0
11 12	2 8 9	5	6	6	17	9·0
11 13	1 11 11	5	7	5	17	8·5
11 16	1 5 27	4	5	...	...	...
11 16	3 7 11	5	5	8	18	10·5
12 3	1 8 12	5	5	6	16	8·5
12 4	2 6 18	4	7	5	16	8·5
12 12	1 11 11	5	7	5	17	8·5

Inferior plates: not full failures.

3-20 D. like	3-18 D.	on photographic chart
5-10	5-9	
9-3	9-1	at λ 447. Too weak at Hζ.
8-10	8-3	„ „
1-16	1-11	
11-17	11-16	at Hζ; but 447 single absorption.
1-18	1-11	
2-18	2-20	
3-20	3 19	
11-19	11-16	
8-15	8-3	at λ 447. Hζ doubtful.

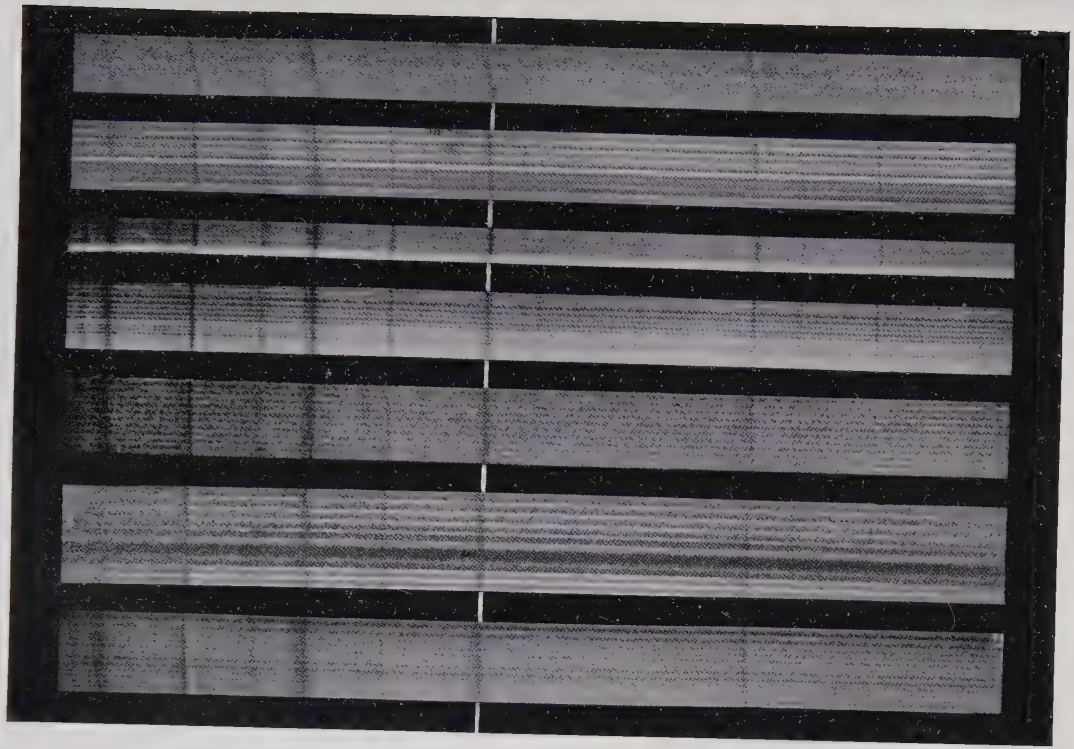


On the Chromatic Correction of Object-glasses. By A. E. Conrady.

The disproportionality of dispersion in crown- and flint-glass renders a perfectly achromatic object-glass an impossibility; a secondary (or in the case of a few special kinds of glass recently introduced by the Jena glassworks, a tertiary) spectrum being unavoidable. With all ordinary combinations of crown- and flint-glass the effect is that light of a particular wave-length is focussed at a minimum distance, while all light of sensibly different wave-length is focussed at a greater distance from the objective. The only control which the optician has over the colour correction, once the glass has been decided upon, consists in choosing the most favourable wave-length for which the

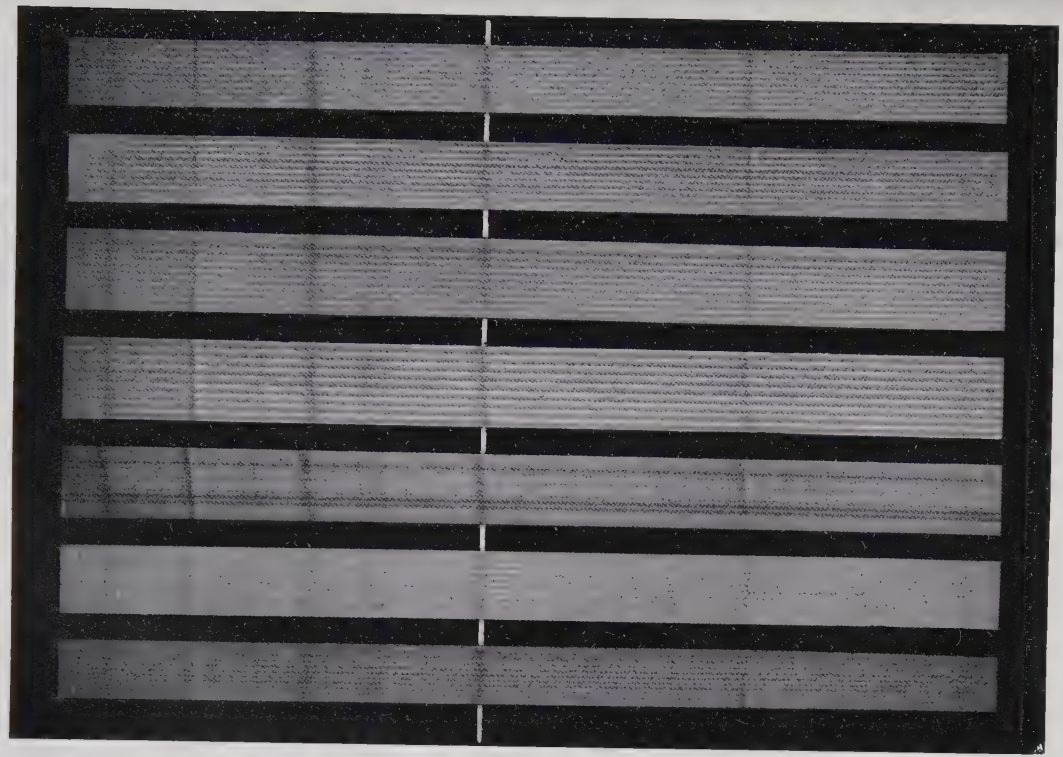
† Rambaut, *Monthly Notices*, vol. l., p. 305.

D
D H
12 15
0 0
0 5
0 6
0 17
1 11
1 14
H_ζ K H_ϵ HELM H_δ
H_γ HELM
447
448
M
Y M D
1902 9 4
1902 5 23
1901 8 25
1902 7 1
1903 7 24
1902 8 9
1902 9 4



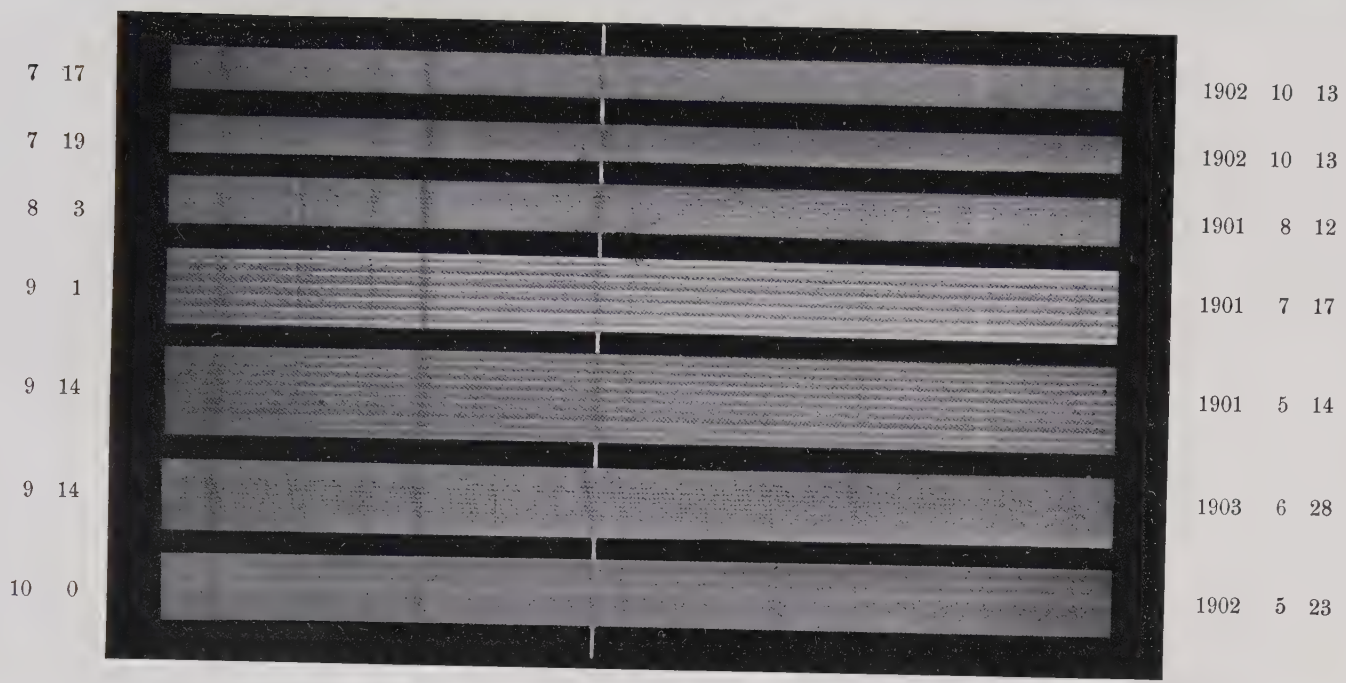
SPECTRUM OF β LYRAE.

D
H$_\zeta$ K H$_\varepsilon$ HELM H$_\delta$
H$_\gamma$ HELM 447 448
M
D H
Y M D
3 12
1901 10 29
3 12
1902 8 22
3 14
1902 9 4
3 18
1901 6 9
3 19
1902 9 30
3 21
1903 8 19
4 6
1902 7 1
2

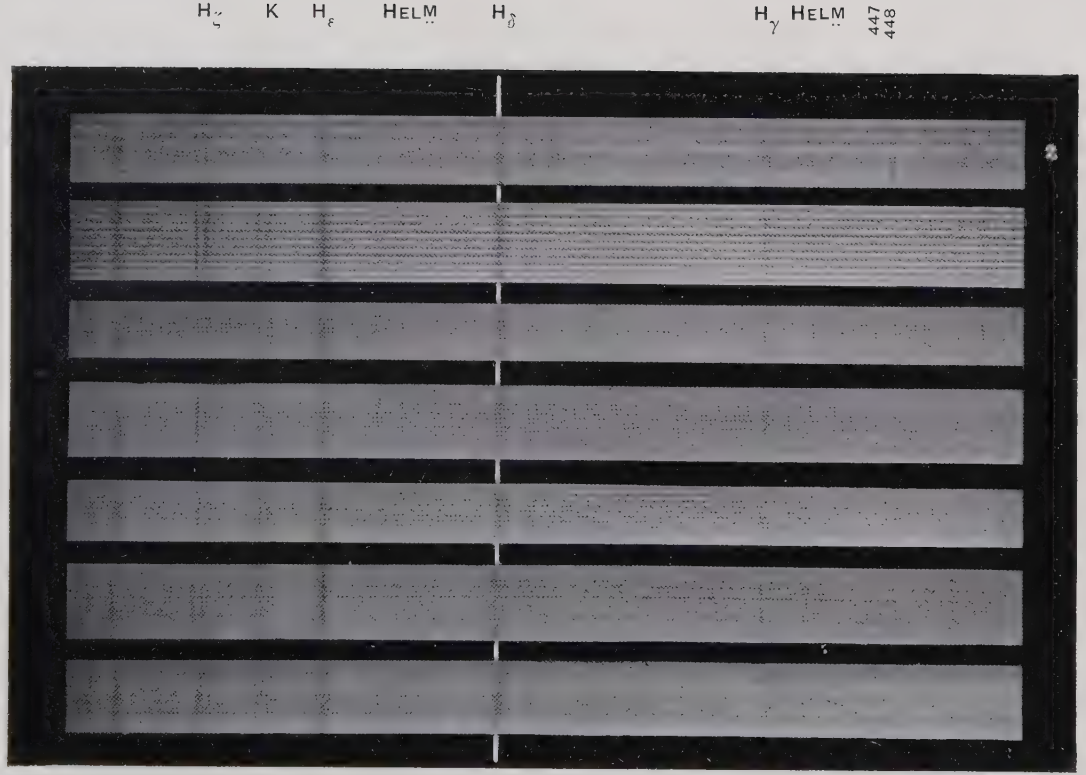


SPECTRUM OF β LYRAE.

D
M
H_ζ K H_ε HELM H_δ H_γ HELM 447 448
D H
4 23
5 9
5 23
6 20
7 4
7 6
7 15
Y M D
1903 9 13
1902 7 27
1903 8 31
1902 10 25
1901 8 12
1901 9 7
1902 9 17



SPECTRUM OF β LYRAE.


D
M
H$_\zeta$ K H$_\varepsilon$ HeIM H$_\delta$ H$_\gamma$ HeIM 447 448
D H
10 0
10 3
10 21
10 23
11 10
11 12
11 13
Y M D
1902 5 23
1901 8 12
1903 8 31
1903 8 31
1902 8 9
1902 8 9
1901 11 11

11	16		1901	5	27
11	16		1903	7	11
12	3		1901	8	12
12	4		1902	6	18
12	12		1901	11	11
12	12		1902	8	22
12	15		1902	9	4

SPECTRUM OF β LYRAE.

objective shall have the minimum focus; and this choice is so important that it is not unusual at the present time to see the selected wave-length stipulated by the astronomer when ordering an object-glass for a particular purpose.

Such a stipulation calls for new methods of computation based on the principles of the differential calculus, as the coloured "rays" to be united must have an indefinitely small difference of wave-length, while the usual trigonometrical computation can be successfully carried out only for widely different wave-lengths.

But it further calls for a mathematical formula connecting the refractive index, n, of the glass used with the wave-length, λ, of light, as otherwise it would be difficult to determine the rate of dispersion for any stipulated wave-length.

Several such formulæ have been proposed which are based on theoretical grounds, but they are too complicated for practical use. Among the empirical formulæ one consisting of the first three terms of Cauchy's used to be a favourite :

$$n_\lambda = n_0 + \nu_1\lambda^{-2} + \nu_2\lambda^{-4},$$

but when this is applied to glass by determining the three constants from the observed indices for three different wave-lengths, the discrepancies between observed and computed values for other wave-lengths are altogether too serious to be tolerated.

A number of years ago a Dr. Willibald Schmidt, by a most laborious series of calculations according to the method of least squares, tried all likely combinations of two *whole-number* powers of λ and came to the conclusion that the formula

$$n_\lambda = n_0 + \nu_1\lambda^{-1} + \nu_2\lambda^{-4}$$

brought about the most satisfactory agreement between observed and computed indices.

But on applying Schmidt's formula to the numerous varieties of glass in the Jena Catalogue, I found decidedly unpleasant discrepancies, smaller than those resulting from Cauchy's, but yet too large, especially for flint-glass. A closer discussion showed that the differences with Schmidt's formula were always of the opposite sign and generally of about one-third the magnitude of those found with Cauchy's formula. And so, after some trials, I adopted an intermediate formula, viz.

$$n_\lambda = n_0 + \nu_1\lambda^{-\frac{2}{3}} + \nu_2\lambda^{-4}.$$

This formula will render the refractive indices of all ordinary kinds of glass correctly throughout the visible spectrum within a few units of the fifth decimal, and is therefore sufficiently accurate for all ordinary purposes. It may confidently be used to interpolate the refractive index for any stipulated wave-length, and, by differentiation, to determine the rate of dispersion in the vicinity of any wave length.

I now proceed to formulate a chromatic condition for use in computing objectives having minimum focus for a prescribed wave-length. From the point of view of the undulatory theory a combination of lenses transforms the spherical waves emanating from a luminous point forming the object into waves converging towards the conjugate point. Obviously a perfectly achromatic telescope would have to transform waves of any wave-length into refracted waves of exactly similar form and consequently of exactly the same focus. With the ordinary kinds of glass and the ordinary types of object-glasses we can attempt this ideal correction for a very short part of the spectrum only, as explained above. But, moreover, we cannot get waves of exactly similar form even within such a narrow range, because such object-glasses do not allow of establishing freedom from spherical aberration for more than one wave-length. Under these circumstances we shall get a good compromise by stipulating that the refracted waves of a wave-length slightly differing from that selected as the principal one shall, when tangent to the principal

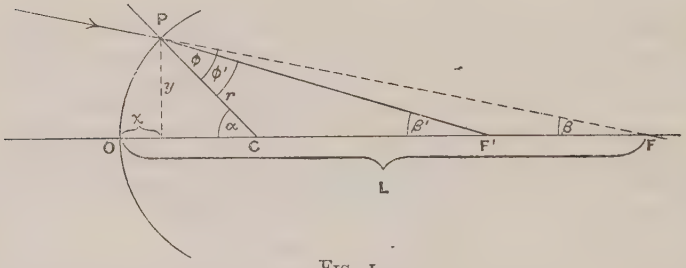


FIG. 1.

wave in the optical axis, also intersect that wave in the margin of the segment which the object-glass allows to pass. It is important to note that the chromatic condition thus formulated does not depend on the spherical correction for the principal wave-length; the formulæ to be given can therefore be applied with equal facility whether a system of lenses is spherically corrected or not.

These formulæ are based on and used in connection with the trigonometrical ones suitable for computing the course of rays through a system of lenses, and as these are not found in ordinary text-books I will give them first. In these calculations a ray, P F in fig. 1, is defined by the distance O F=L from the vertex of a refracting surface at which it cuts the optical axis—positive on that side of the surface towards which the light travels—and the angle β under which it cuts the optical axis—positive when the ray, in crossing, passes from above to below the axis. The sign of the radius of curvature r is determined by the same rule as that of L.

Calling the angle of incidence CPF ϕ, the triangle CPF gives

I. $$\text{Sin } \phi = \sin \beta \frac{\text{CF}}{\text{CP}} = \sin \beta \frac{\text{L} - r}{r}.$$

Next we obviously have the angle α at the centre of curvature.

II. $$\alpha = \beta + \phi.$$

Distinguishing the angles and coordinates after refraction by a dash, we have, from the law of refraction, when n is the index of the medium to the left, n' that of the medium to the right of our refracting surface.

III. $$n \sin \phi = n' \sin \phi' \quad \text{or} \quad \sin \phi' = \frac{n}{n'} \sin \phi, \text{ whence } \phi'.$$

The new triangle CPF′ next gives

IV. $$\beta' = \alpha - \phi',$$

and further, in analogy with I.,

V. $$\text{L}' - r = r \frac{\sin \phi'}{\sin \beta'}, \text{ whence L}'.$$

We thus have the coordinates of the refracted ray and can at once proceed to any following refracting surface. We can also find the rectangular coordinates of point P; they are

$$x = \text{R}(1 - \cos \alpha) = 2\text{R} \sin^2 \frac{\alpha}{2} \text{ and } y = \text{R} \sin \alpha.$$

Returning to our problem, we must bear in mind that light travels the slower the denser the medium, taking the same time to traverse a distance d in a medium of refractive index n as a distance $n \cdot d$ in empty space. $n \cdot d$ is called the equivalent "optical path" of the distance d in medium of index n; any path may be reduced to its equivalent optical path by multiplying each portion by the index of the medium in which it lies.

Now, in any system of lenses the optical paths of all rays of any one wave-length connecting the waves sent out by a luminous point with the same waves after passing through the system are necessarily equal; in other words, light of any one wave-length will, owing to the retardation in glass, travel in the same time over the axial distance between two such waves as over the geometrically longer marginal distance. My chromatic condition stipulates that the axial and marginal paths shall also be optically equal between the same points for light of a slightly different wave-length.

It is obvious that a coloured ray of slightly different wave-length will follow a slightly different path from that taken by

the principal ray; we shall have to ascertain whether this difference is sensible. In fig. 2 let the thick line $r\,r'$ represent the principal ray, the thin line $\rho\,\rho'$ a coloured ray of slightly different wave-length. The path of the latter will form small angles with and be separated by small distances from that taken by the principal ray, and, the law of refraction being of a linear character, these angles and distances are of the same order of magnitude as the difference in refractive index. The angle between the two paths will make that of the coloured ray longer in proportion to its secant; as this differs from unity only by a term of the *second* order we may neglect this factor. We next notice that the path of $\rho\,\rho'$ is alternately longer and shorter than that of $r\,r'$; but here again there is compensation at each

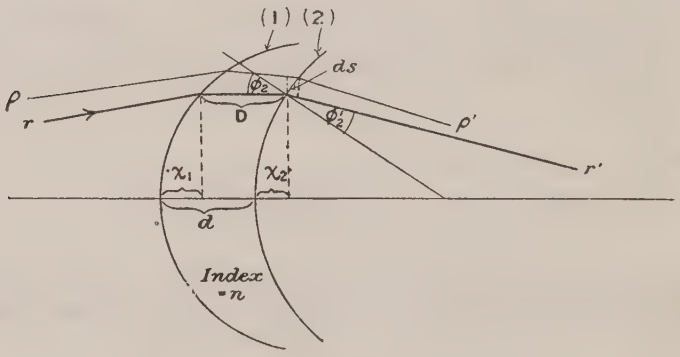


FIG. 2.

refracting surface; for, taking as an example the surface (2), we have obviously

The path of the principal ray within surface (2) shorter by $ds \,.\, \sin\phi_2$ or *optically* by $ds \,.\, n \,.\, \sin\phi_2$,
but the path of the principal ray beyond surface (2) longer by $ds \,.\, \sin\phi'_2$ or *optically* by $ds \,.\, n' \,.\, \sin\phi'_2$;

and as, by the law of refraction, we have $n \,.\, \sin\phi_2 = n' \sin\phi'_2$ there is complete compensation as far as first order terms are concerned.

Hence we may assume without error that the coloured rays (of an indefinitely small difference in wave-length) follow the same course as the principal ray, which greatly simplifies the problem. For if we call the geometrical paths of the principal axial and marginal rays d and D respectively, the refractive index for the principal ray n, that for the coloured ray $(n + dn)$, we shall have, in accordance with the explanations given above,

$$\Sigma\, n(d - D) = 0$$

when this sum is taken over the entire path from a wave sent out by the object to the same wave after refraction.

Similarly, the total difference of the optical paths of the coloured rays will be

$$\Sigma (n + dn)(d - \mathrm{D}),$$

or, deducting the preceding equation,

$$\Sigma\, dn\, (d - \mathrm{D}),$$

and the system of lenses will be achromatic for the selected wave-length when this sum is made equal to zero; hence the mathematical formula expressing my new chromatic condition is

$$\text{VI.} \qquad \Sigma\, dn\, (d - \mathrm{D}) = 0.$$

Here the dn are to be found as explained in the early part of this paper. The d are given. The D are easily determined by projection upon the optical axis, which gives

$$\mathrm{D} = (d + x_2 - x_1) \sec \beta'_1$$

$$\text{or } \mathrm{D} = \left\{ d + 2r_2 \sin^2 \frac{\alpha_2}{2} - 2r_1 \sin^2 \frac{\alpha_1}{2} \right\} \sec \beta'_1.$$

As, in air, we have practically $n = 1$ and $dn = 0$, the D have to be computed only for those portions of the optical system which consist of glass or some other dense medium.

This formula is so simple and convenient in practical use that I always use it, even in cases when the older method of computing trigonometrically for two widely different wave-lengths would be equally correct. One further important advantage afforded by it is that it greatly reduces the number of trials necessary to obtain a properly corrected lens. For it easily yields a rigorous formula by which the last radius of a system of lenses may be at once determined so as to get perfect achromatism. With the older method even this has to be attained by successive approximations.

In ordinary cases it is not always necessary to obtain the dn by differentiation. Thus for visually corrected objectives it is generally sufficient to take the dispersion between the C and F lines as the value of dn, and similarly for ordinary photographic objectives the dispersion between the D and G lines may be used in a like manner.

I must add that the formulæ given are also applicable to paraxial rays by proper modification.

Taking the trigonometrical formulæ first, we note that for rays very close to the optical axis all the angles become very small, hence equal to their sines. Distinguishing these small paraxial angles by a left-hand index o and the corresponding

abscissæ of the paraxial rays by small letters l, we can write the formulæ down at once :

$$_{0}\text{I.} \quad -{}_{0}\phi = {}_{0}\beta\,\frac{l-r}{r}$$

$$_{1}\text{II.} \quad {}_{0}\alpha = {}_{0}\beta + {}_{0}\phi$$

$$_{0}\text{III.} \quad n\,.\,{}_{0}\phi = n'\,.\,{}_{0}\phi' \text{ or } {}_{0}\phi' = \frac{n}{n'}\,.\,{}_{0}\phi$$

$$_{0}\text{IV.} \quad {}_{0}\beta' = {}_{0}\alpha - {}_{0}\phi'$$

$$_{0}\text{V.} \quad l'-r = r\,\frac{{}_{0}\phi'}{{}_{0}\beta'}$$

To determine the chromatic condition for paraxial rays, we first transform the term $(d-\text{D})$ in VI. by introducing the value of D given by VII., and obtain

$$d-\text{D} = d - \frac{d + 2r_2 \sin^2\frac{\alpha_2}{2} - 2r_1 \sin^2\frac{\alpha_1}{2}}{\cos\beta_1'}$$

$$= \frac{2r_1 \sin^2\frac{\alpha_1}{2} - 2r_2 \sin^2\frac{\alpha_2}{2} - d\,(1-\cos\beta'_1)}{\cos\beta'_1}$$

$$= \frac{2r_1 \sin^2\frac{\alpha_1}{2} - 2r_2 \sin^2\frac{\alpha_2}{2} - 2d \sin^2\frac{\beta'_1}{2}}{\cos\beta'_1}$$

If now the angles become very small, the cos in the denominator will become equal to unity, and we shall have

$$d-{}_{0}\text{D} = \tfrac{1}{2}\{r_1\,.\,{}_{0}\alpha_1{}^2 - r_2\,.\,{}_{0}\alpha_2{}^2 - d\,.\,{}_{0}\beta'_1{}^2\}$$

and the condition of achromatism for paraxial rays will be

$$_{0}\text{VI.} \quad \Sigma\, dn\,(d-{}_{0}\text{D}) = 0.$$

It will be clear that the fulfilling of this condition will give refracted coloured waves of the *same radius of curvature* as that of the principal waves ; in fact, this condition gives the identical result as the old one of equal focal point for principal and coloured paraxial rays.

When the conditions stated by VI. and $_{0}$VI. are fulfilled simultaneously, we attain a very high state of chromatic correction (semi-apochromatic correction) practically but not absolutely the same as that resulting from the well-known Gauss-condition.

Note on the use of Long-focus Mirrors for Eclipse work.
By H. H. Turner, F.R.S., Savilian Professor.

1. At recent eclipses the inner corona has been successfully photographed on a large scale with long-focus lenses : lengths of 40 feet, 60 feet, and even 100 feet do not seem to be unmanageable. Such lenses are naturally costly, and it is of interest to inquire how far they can be replaced by concave mirrors, which are cheaper.

2. It may be taken for granted that any telescope of such a focal length must be used in conjunction with a cœlostat, or some form of heliostat which allows the telescope to remain fixed in position. We may therefore disregard at present any disadvantages which are introduced by the cœlostat itself, and exist whether we use a refracting or reflecting telescope—such, for

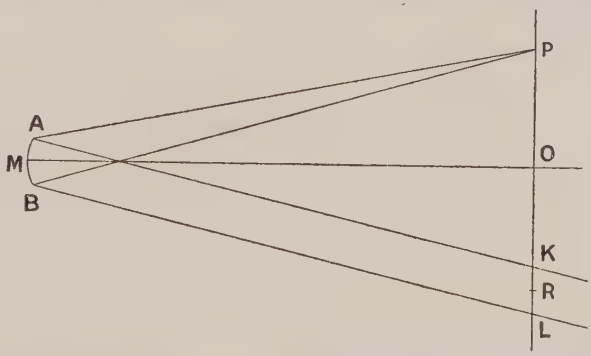


Fig. 1.

instance, as the want of perfect flatness in the plane mirror. But there is an important difference between the two cases in respect of the available field, which it is the object of the present Note to briefly elucidate. Practically there is no difficulty about the field with a lens, unless it be placed unnecessarily far away from the cœlostat (see *Monthly Notices*, lvi. p. 413).

3. As regards the mirror. Let A B (fig. 1) be the concave mirror, centre M and axis M O ; and P any point in the image formed in the focal plane. If we trace the rays backwards from P to the mirror, and then towards the origin of light, they will form a sensibly circular cylinder which cuts the focal plane in a circle K L sensibly equal to the mirror, and having its centre at R, where O R = O P. For ideal definition at the point P this cylinder of rays must all fall on the cœlostat mirror, otherwise only a portion of the concave will be used in forming the image at P. But in practice we shall be able to put up with something

short of this ; how much short of it remains to be determined, probably by actual experiment.

4. As we move P about to different portions of the image, R moves about over a precisely similar path, carrying the circular patch K L with it. And for ideal definition at all points of the image, the total area thus indicated must fall on the cœlostat mirror. Thus we can find the size and shape of the smallest cœlostat mirror which must be used in the focal plane, when we know the contour of the image. *Or conversely*, if we know the size and shape of the mirror we can find the contour of the image within which there will be ideal definition.

5. It is more convenient to consider the problem from this latter point of view, for the effective shape of the mirror (*i.e.* its projection on the focal plane) will usually be an ellipse. For a circular mirror of radius a placed near the focus of the concave, and inclined at an angle θ to the focal plane, the axes of this ellipse are $2a$ and $2a \cos\theta$; or, say, $2a$ and $2b$. In fig. 2 let X Y be this ellipse, and let a circle with centre R and radius c equal to

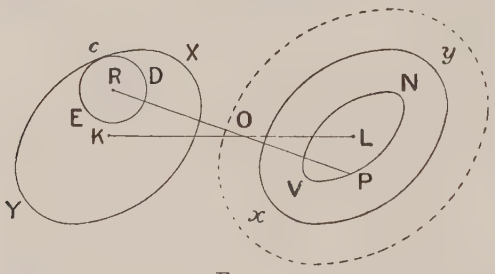


FIG. 2.

the radius of the concave mirror touch X Y internally. As R moves about, keeping this circle in contact with the ellipse, its image P will describe the contour of the image formed by the whole concave, and it is easily seen that this contour is an oval curve of axes $2(a-c)$ and $2(b-c)$. Thus unless the cœlostat mirror is so large that

$$a \cos\theta > c$$

then no part of the image will get light from all parts of the concave. But we can of course always cut down the concave by a diaphragm to secure this condition.

6. If we are satisfied with the rays from about *half* the concave mirror, then we can always get an available image equal in size and shape to the ellipse X Y. For if R lie on the edge of X Y, half the circle surrounding it (a little more indeed) will fall outside X Y, but the remaining portion will lie within ; and the image formed at P will still be as good as (say) a heliometer image, which uses half the object glass. As R moves round the edge of X Y, P will describe a precisely similar ellipse

xy ; and within this ellipse, which is equal in size and shape to the projection X Y of the cœlostat mirror, we accordingly get a fair image. A larger oval found by making R move so that the circle CDE touches X Y externally gives the limit of any image at all.

7. To fix the ideas, suppose that the cœlostat mirror is 16 inches in diameter and is inclined at an angle of 45° to the incident beam ; so that $2a = 16$ inches, $2b = 11{\cdot}3$ inches. Then with a concave of 12 inches diameter we shall get no field at all filled by the whole mirror; but if we cut the aperture of the concave down to 8 inches we get an oval field 8 inches long and 3·3 inches wide filled by the whole mirror. If we do not cut the 12-inch mirror down we get *more light* at every point of the image, but this light comes from a non-circular mirror. It is a question which can be worked out theoretically, but will probably be more easily settled experimentally, how far the extra light got from a large concave is a gain ?

8. One point seems clear. If we place the image of the eclipsed Sun centrally we shall lose the best part of the image in the central part obscured by the Moon. To make clear what is meant by central let us recur to fig 2. The important axis for us is the line joining K, the centre of the cœlostat, to the centre of the concave mirror, which cannot be represented in this figure—call it M, as in fig. 1. It is the line K M which forms the central ray, reflected along M L to the centre of the image. The normal M O to the concave mirror is only of secondary importance. Keeping M fixed, we may tilt the concave mirror and thus move O about to any point in the plane represented by fig. 2, carrying the point L and the patches surrounding it to positions represented by the law of images, *i.e.* K O = O L always. And if the dark centre of the Sun is viewed from M in the cœlostat in the direction of MK, the image of the dark centre will be formed at L, the best part of the field. Hence we must clearly arrange to see in the direction MK a part of the corona, which can be then photographed at L.

9. Before leaving the concave, remark that we cannot pass from one part of the corona to another by tilting it. Whatever part can be seen from M in the cœlostat (in the direction M K) is reflected to L, which should be the centre of the plate ; and by tilting the concave we merely select a different position for putting the plate. At the same time we alter the angle of incidence on the concave of the field we are photographing ; and since it is important to have this as small as possible, the proper tilt of the concave is immediately indicated—viz. it should be such that the line KOL is in the direction of the minor axis of X Y : and O should be as near to K as is possible without cutting off any portion of the image which it is wished to keep. When the concave is fixed in accordance with these conditions, the proper position for the plate is also fixed.

10. But we may, during the eclipse, alter the position of the cœlostat slightly in R A, and thus bring different parts of the corona into the field. We will consider this point presently.

11. Let us first get some sort of an idea how much of the corona can be photographed at once. The diameter of the Moon with different focal lengths is as follows :—

Diameter in inches	8	12	16
Focal length in feet	69	104	138

Thus even with a concave of 140 feet focus we could, with a 16-inch cœlostat, photograph about half the corona at once, if we are content with light from about half the concave mirror over some portions of the image. The definition will vary in quality in different parts of the image, the amount of variation depending on the size of the concave mirror. With shorter focal lengths we get more of the corona at a single exposure; with longer focal lengths less. But as the focal length increases the angle of incidence diminishes, and the portion which is photographed will therefore be better photographed.

12. Thus with a concave of 140 feet focus and a 16-inch cœlostat we could bring one limb of the Sun central during the early part of totality and take photographs of it; then, using the slow motion of the cœlostat in R A, bring the other limb central and photograph that. Note that the cœlostat is close to the observer and he could easily manage to do this, or to superintend it, and the position for plates is not altered. The two limbs would not necessarily be either the R A or N P D limbs, being determined entirely by the direction of the minor axis of the ellipse which represents the cœlostat mirror. Portions of the limb 90° away from these would suffer most from defective definition and loss of light.

13. It seems to me possible, though I have not at the moment time to examine the point, that these portions of the corona might be brought central by tilting the cœlostat mirror in N P D—*i.e.* inclining it slightly to the polar axis. Such a tilt is of course an error in adjustment, and produces a slow change in the image, but the change might not be serious enough to matter under the circumstances of an eclipse.

14. One small point may be noticed in conclusion. If such mirrors are used, the photographs taken will not be suitable for determining the relative brightness of different parts of the corona without a troublesome calculation of the amount of light received by different parts of the image.

On Graphical Methods of determining the Local or Greenwich Time of Sunset at different places within a given region. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. A few years ago my attention was directed in two very different ways to the problem of determining by some simple method—preferably a graphical one—the time of sunrise or sunset at various places within a limited region. First, Mr. Robert Sewell, in discussing Indian eclipses, wanted a correction for reducing the time of sunset or sunrise at any place in India to the time of sunset at Lanka. Secondly, it was decided that cyclists were to be guided in lighting their lamps by the time of local sunset, and not by that of sunset at Greenwich. It was easy to devise *some* graphical method, but it was not so easy to arrive at one which seemed to be final, for new, though slight, simplifications were apt to suggest themselves. In 1902 August I had arrived at the form described below, and no modification of value has since occurred to me; but in 1903 September, during the visit of the British Association to Belfast, I found that Mr. D. E. Benson and Mr. G. Napier Clark, who had also been working at the problem for some years, had arrived by a different route at a somewhat similar solution, of which a brief description is appended.

2. The ordinary processes involve two corrections, one for longitude and one for latitude. The former is a mere addition or subtraction; the latter is made usually by spherical trigonometry, put into the form of tables perhaps. To combine the two, remark that the places where the Sun is setting at any instant lie on a great circle of the Earth, if we neglect refraction; or in a small circle parallel to it at a distance of about 33′ if we take in refraction. As the Earth rotates, this circle travels over its surface, maintaining constant its position relatively to successive meridians.

3. Thus if we take any map on which the meridians are represented by parallel lines at equal distances for equal differences of longitude, and draw the sunset-line for any moment, its position for any other moment is obtained by simply sliding the sunset-line uniformly across the map in a direction perpendicular to the meridians. Thus, in fig. 1, if AB be the sunset-line on any date at Greenwich sunset, then by sliding it a horizontal distance aA, so as to pass through Hull, we obtain the sunset-line at Hull sunset, and the distance Aa measures the elapsed time.

4. But the sunset-line will on most maps be a curved line, excepting at an equinox, when it coincides with a meridian, if refraction be neglected. (If refraction be included there will generally be some date near an equinox for which the sunset-line is straight.) We can choose the form of the map so that the

sunset-line will be straight at some one particular date, for we can set our lines of latitude on any arbitrary scale, but at other dates the lines will still be curved. As this point is important we will examine it in detail before proceeding further.

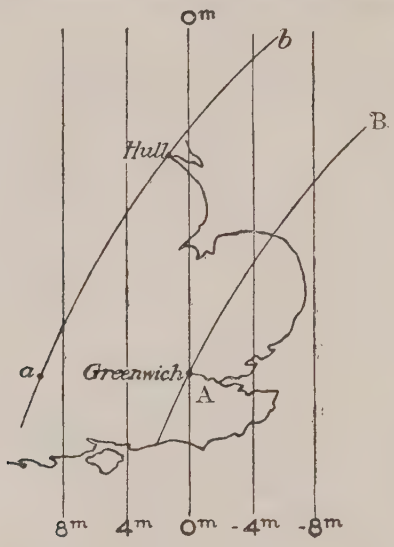


FIG. 1.

5. Neglecting refraction, the hour-angle of the Sun at setting differs from 90° by an angle

$$\theta = \sin^{-1}(\tan\phi \tan\delta)$$

where δ is the Sun's declination, and ϕ the latitude of the place. The hour-angle represents local time. If l be the longitude of the place, the G.M.T. of sunset will be

$$l+\theta$$

where l and θ are both expressed in time.

Since l increases uniformly in one coordinate, θ must increase uniformly in the other if we are to have a straight line ; and thus we must lay off our latitudes so that

$$\sin^{-1}(\tan\phi \tan\delta)$$

increases uniformly. Now it is easily seen that if this is true for one value of δ (not zero) it will not be true for another.

6. But when we are dealing with a limited area—such as Great Britain, say—we can make all the lines *sensibly* straight, since δ is never greater than $23\frac{1}{2}°$. In illustration of this point let us make the lines straight when $\delta = 26° 36'$ and see what the de-

viation is when $\delta' = 14° 5'$, which should give about the maximum error, remembering that $\delta = 0°$ is always straight. These values of δ are chosen because $\tan 26° 36' = \cdot 5$ and $\tan 14° 5' = \cdot 25$, which makes the multiplications $\tan\phi \tan\delta$ easy. In Table I. column A gives the values of θ for $\delta = \tan^{-1} 0\cdot 5$, and column B the values of θ for $\delta = \tan^{-1} 0\cdot 25$. By comparing the two we find that, *approximately*,

$$B = 4°\cdot 4 + \cdot 359\ A$$

and the values calculated from this formula are given in column C. The difference B−C never exceeds $0°\cdot 2$ or $0^m\cdot 8$, which may perhaps be neglected for our present purpose.

Table I.

ϕ	A.	B.	C.	B−C.
°	°	°	°	°
50	36·6	17·3	17·5	−0·2
52	39·8	18·7	18·7	0·0
54	43·5	20·1	20·0	+0·1
56	47·8	21·8	21·6	+0·2
58	53·2	23·6	23·5	+0·1
60	60·0	25·7	25·9	−0·2

Thus between parallels 50° and 60° we can use straight sunset-lines without any great error if we lay off our parallels of latitude so as to make the extreme case of $\delta = 27°$ straight. The error will be much less if we make a map of England alone, which extends from 50° to 56° only.

7. We have taken limits 50° to 60° of latitude in the above example. In Table II. we see that the proposition is very nearly true from *the equator* up to 60° for values which δ can actually assume. Taking $\tan\delta_1 = \frac{1}{3}$ (or $\delta = 18\frac{1}{2}°$ about) as the case for actual straightness, the greatest deviations from straightness will occur when $\tan\delta_2 = \frac{1}{6}$, say, and $\tan\delta_3 = \cdot 44$ ($\delta_3 = 23\frac{1}{2}°$). Column D gives the value of θ for δ_1; column E for δ_2, which is compared with ·49 D; and column F for δ_3, which is compared with 1·35 D.

Table II.

ϕ	D.	E.	E−·49 D.	F.	F−1·35 D.
°	°	°	°	°	°
0	0·0	0·0	0·0	0·0	0·0
10	3·4	1·7	0·0	4·4	−0·2
20	7·0	3·5	+0·1	9·2	−0·2
30	11·1	5·5	+0·1	14·7	−0·3
40	16·3	8·1	+0·1	21·6	−0·4
50	23·4	11·5	0·0	31·5	0·0
60	35·3	16·8	−0·5	49·6	+1·9

The extreme error in the last case is 1°·9, or nearly 8^m, which can scarcely be neglected; but it is evident that it is only near 60° that the error becomes large.

8. But now returning to a special problem, such as a sunset diagram for England, we have established this proposition.

When meridians are represented on a map by equidistant parallel lines it is possible to choose a variable scale of latitude so that the sunset-lines at all dates are straight within limits which we may neglect.

The proposition has been examined, for simplicity's sake, when refraction is neglected; but since refraction only slightly modifies the lines, it is readily seen to be also true when refraction is taken into account.

9. To make the best latitude scale for the purpose we may proceed as follows:—

Select the date for which $\tan\delta = \frac{1}{3}$ ($\delta = 18\frac{1}{2}°$) as that for which to make the line accurately straight, or some date near this. We shall get the maximum deviations from straightness at about $\delta = 9°$ (since $\delta = 0°$ is also straight) and $\delta = 23\frac{1}{2}°$, as in Table II.

Calculate the accurate times of sunset, including refraction, for each degree of latitude on this date, and let them be as follows:—

Latitude	50°	51°	52°	53° . . .
Local time of sunset	t_0	t_1	t_2	t_3 . . .

Then the space between 50° and 51° is to be proportional to $t_1 - t_0$; that between 51° and 52° to $t_2 - t_1$; and so on. This enables us to set out our latitude-lines.

FIG. 2.

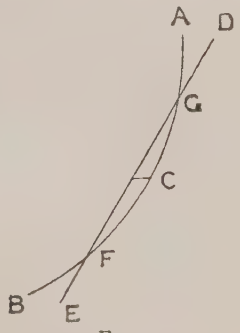


FIG. 3.

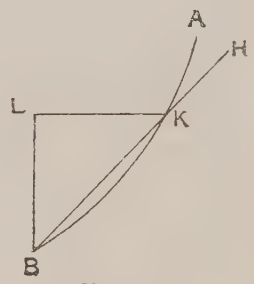
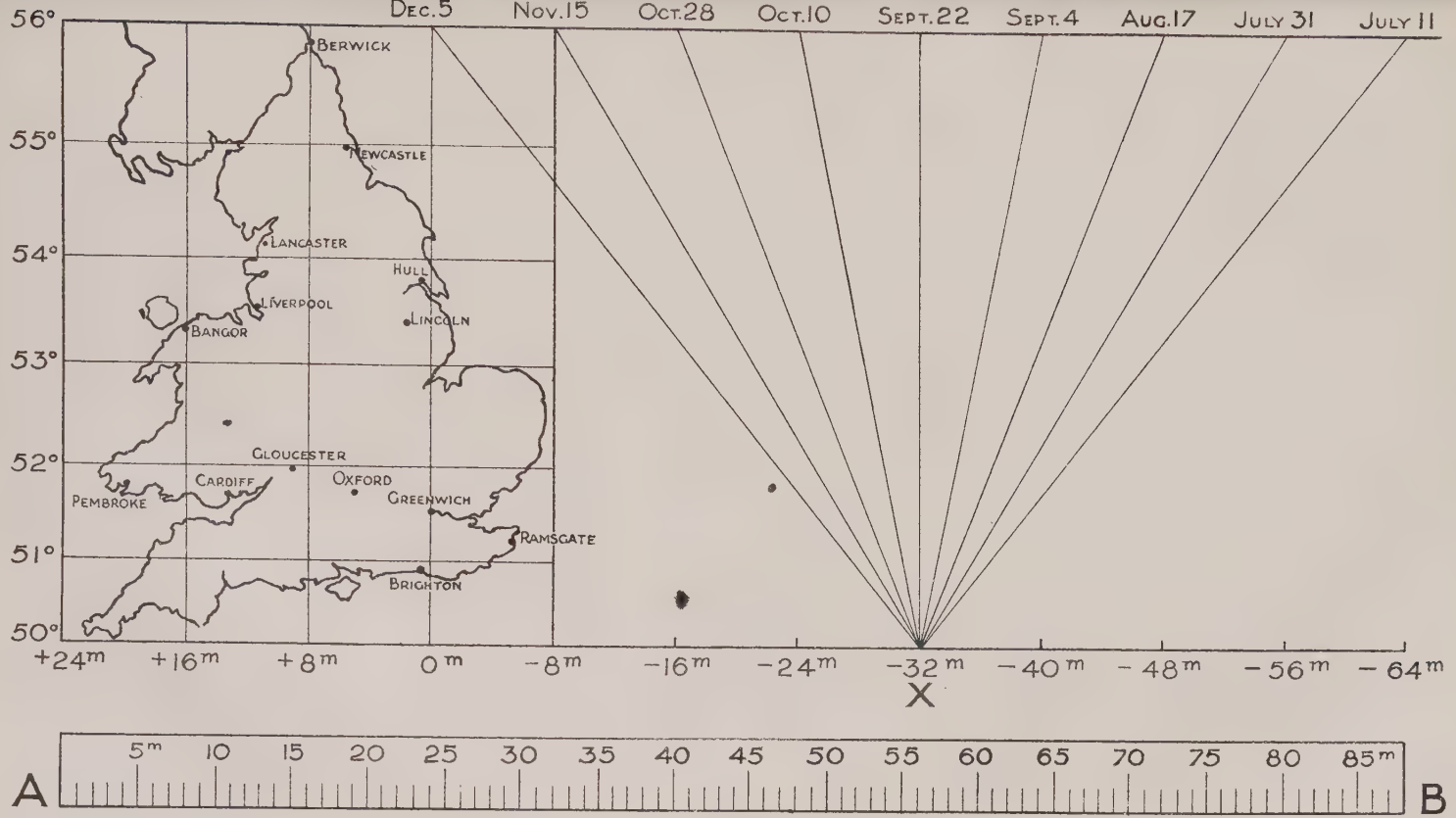


FIG. 4.

10. To find the sunset-line for any date.—We are really representing a curved line by a straight one, and can make the compromise in a variety of ways, none of them deviating much from the truth, for the straight line and the curve will cut at two points which may be varied in position. If we take these


Dec.5
Nov.15
Oct.28
Oct.10
Sept.22
Sept.4
Aug.17
July 31
July 11
56°
55°
54°
53°
52°
51°
50°
Berwick
Newcastle
Lancaster
Hull
Liverpool
Lincoln
Bangor
Gloucester
Cardiff
Oxford
Greenwich
Pembroke
Ramsgate
Brighton
+24m
+16m
+8m
0m
−8m
−16m
−24m
−32m
−40m
−48m
−56m
−64m
X
A
5m
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85m
B

points of intersection at the top and bottom of the map, as in fig. 2, the error will be greatest in the middle, and always of one sign. It will be better to put the points of intersection, as in fig. 3, so that DG = EF = $\frac{1}{6}$ DE, and then we shall distribute the error. But if we are drawing a series of lines from a fixed point B the best compromise may be BKH, as in fig. 4. The general principle of getting a good compensation will be obvious from these illustrations. In practice the rule comes to this: Select two definite latitudes, either those of F and G or those of Band K, as the case may be, and find the difference in time of sunset for these two latitudes at any date. If BL be a meridian, set off LK equal to this difference on the latitude-line through L; then BK will be the sunset-line for the date.

11. We have now considered everything but the *size* of the map, which is an important practical detail, and determines the scale of time. We may represent the G.M.T. of sunset in two ways:

(*a*) Either the map shall show the actual G.M.T. of sunset, or

(*b*) It shall show the *difference* which is to be added to (or subtracted from) the time of sunset at Greenwich, which can be given in a separate table.

12. If we adopt (*a*), then, since the time of sunset at Greenwich itself varies more than four hours, and that at Berwick more than five hours, with a map 6 inches long we can only allow 1 inch to an hour. If we adopt (*b*), the whole *difference* to be represented is only about 80 minutes, so that we can have nearly 4 inches to the hour. It is true that method (*b*) involves adding two quantities together instead of taking a single quantity from the map; but it seemed to me that the advantages of an open scale were too great to be neglected.

13. Having settled the scale of longitude, that of latitude follows generally from the consideration that the extreme sunset-lines should be inclined about 45° to the meridians—not more, and preferably less.

14. I will now give an indication of the form which I had arrived at for the map last year (see Plate 12).

Method (*b*) being adopted, it was determined to represent the difference of sunset-time *not* from that at Greenwich, which involves + and − signs, but from a point X, 32^m east of Greenwich, and in latitude 50°, so that the correction to sunset at X should always be additive.

The sunset-lines are represented by straight lines through X, the times of sunset being accurately calculated for the parallel 54°.

The scale of latitude expands as we go north, being accurately proportional to the differences of sunset-time for tan $\delta = \frac{1}{3}$.

A certain number of the lines are drawn corresponding to dates shown at the top; those for intermediate dates can be

inferred. The other half of the year is shown on another scale of dates, which can be folded down over that represented in the figure.

Supplementing the map there must be given a table of times of sunset for the point X.

To find the time of sunset at any point P for any date, take the paper scale AB (which is a scale of minutes and should be detached from the map), lay the corner A on P, keeping the scale horizontal, and read where it cuts the date-line. This reading gives the number of minutes to be added to the time of sunset at X, which is given in the table.

15. The scheme of Mr. Benson and Mr. Clark may be indicated as follows :—

(*a*) They take Mercator's projection for the map. The latitude scale is a gradually expanding one, and happens therefore to be nearly the same as that we were led to theoretically in what precedes. Mr. Benson and Mr. Clark determined its suitability for straight sunset-lines by experiment.

(*b*) They place the reference point X in the centre of the map, which involves the use of + and — signs, but reduces the size of the map.

(*c*) The sunset-lines are not actually drawn. The dates are indicated at the top and bottom of the map, and a string fastened at X can be stretched to the date so as to form any required sunset-line.

(*d*) No separate scale AB is given. The distance of a place to right or left of the line is to be inferred from the distances between meridians. This is indeed not difficult when the point X is placed in the middle of the map, and the distances are all small.

(*e*) The table giving sunset at X is practically ranged alongside the dates given at the top and bottom of the map, in the form of an accompanying time-scale. This is in many ways convenient, and the only disadvantage is (as indicated in § 11) that the scale is rather small, as four hours are to be represented within the dimensions of the map.

Tables to facilitate the working of Combined Altitudes by Saint-Hilaire's Method. By Lieut. E. B. Simpson-Baikie, R.N.R.

Communicated by the Secretaries.

The most valuable of all methods for finding a ship's position at sea is no doubt the combined altitude method, which consists of either a combination of simultaneous observations of different heavenly bodies, or a combination of successive observations of the same one.

There are several systems of utilising such observations, according to the positions of the bodies. When a star is near the prime vertical, the hour angle and thence the longitude may be calculated from the altitude, declination, and assumed latitude; when near the meridian, the latitude may be found by the "ex-meridian" method.

In Marcq Saint-Hilaire's method the position of the observer is assumed, the altitude of the body calculated for the instant of observation from this assumed position, and compared with the true altitude.

In all methods the combination of the two or more observations is effected by drawing "lines of position" at right angles to the lines of azimuth of the observed bodies through the approximate positions resulting from each observation on a Mercator chart. These "lines of position," being short, may be considered arcs of circles of altitude of the respective bodies, and therefore geometrical "loci" for the "true position," which lies at their intersection.

To ensure accuracy the altitudes should not exceed 80°, and the angular difference (difference in azimuth) between two bodies should not be less than 25° or greater than 155°.

In Saint-Hilaire's method the "lines of position" are displaced as many miles towards (or from) the body as the number of minutes of arc that the true altitudes are greater (or less) than the calculated altitudes.

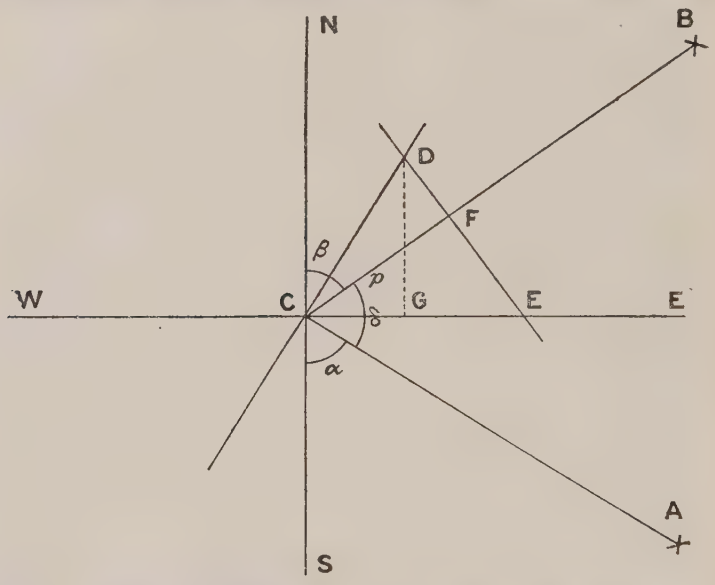
The great advantage of this method over all others is that it may be used with observations of heavenly bodies in any position in azimuth, with only the two restrictions mentioned above, which are common to all methods of combined altitudes. This advantage has been recognised in the Imperial German Navy, where Saint-Hilaire's method (*Höhenmethode*) is now taught almost to the exclusion of all others. In the Royal Navy the junior ranks are instructed in it, and it is a compulsory subject in the course for first-class navigators.

One disadvantage still exists. This is the necessity of obtaining the final result graphically from a chart or sheet of paper—a proceeding which is both clumsy and tiresome. The alternative of calculating the corrections to the assumed position is slightly confusing and somewhat lengthy.

To remove this disadvantage tables have now been computed by which the difference of latitude and difference of longitude between the intersection of the "lines of position" and the assumed position may be found and a final result obtained with few additional figures. These tables are founded on the following considerations, which are explained in "Brent, Walter, and Williams' Ex. Mer. Alt. Tables."

In fig. 1 let NS represent the meridian and WE the parallel of latitude passing through C, the "assumed position." A is one star, whose azimuth is a and whose true and calculated altitudes agree. CD is the line of position due to the observation

of A. B is another star, whose azimuth is β. Let the difference in azimuth between A and B be δ. The difference between the true and calculated altitudes of B is CF $= p$. The



line of position FE, determined by the second observation, cuts the first line of position at D, the "true position."

Now, $$DC = \frac{DG}{\sin DCG}$$

I. $\therefore DG = DC \cdot \sin DCG,$

also $$DC = FC \,.\, \sec DCF.$$

II. $$DG = FC \,.\, \sec DCF \sin DCG,$$

but, $$DCG = ACS = \alpha,$$

and $$DCF = (\alpha + \beta) - 90°$$

$$\therefore \sec DCF = \operatorname{cosec} 180° - (\alpha + \beta) = \operatorname{cosec} \delta.$$

Therefore for II. we put: correction lat. $= \frac{p \,.\, \sin \alpha}{\sin \delta}$.

Similarly: $$CG = DC \,.\, \cos DCG$$

$$CG = FC \,.\, \sec DCF \,.\, \cos DCG$$

$$\therefore \text{ correction departure} = \frac{p \,.\, \cos \alpha}{\sin \delta}$$

And, correction longitude $= \frac{p \,.\, \cos \alpha}{\cos \text{lat} \,.\, \sin \delta}$.

The expressions $\frac{p \cdot \sin a}{\sin \delta}$ and $\frac{p \cdot \cos a}{\sin \delta}$ are so related as to make it possible, p being taken as unity, to construct a table with values of a as arguments at the head of the columns and the values of the corresponding complements of a at the foot, each line corresponding to successive values of δ. The table so constructed gives values of a from 0° to 90° for every whole degree, and values of δ from 20° to 70° for every whole degree, and from 70° to 90° for every alternate degree. When δ exceeds 90° its supplement must be used for entering the table.

To prevent error it must be remembered that the arguments for finding the corrections in latitude and longitude due to the difference between the true and calculated altitudes of a star B (see diagram) are the difference in azimuth (δ) between the two stars, and the azimuth (a) of the *other* star, A.

An additional table gives the values of difference of longitude corresponding to departure within suitable limits.

Observations of Occultations of Stars by the Moon made at the Royal Observatory, Greenwich, in the year 1903.

(*Communicated by the Astronomer-Royal.*)

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1903. May 2	Disapp. 68 Geminorum	Great Equatorial	670	Dark	7 23 38·18	B.
June 2	,, p^3 Leonis	Merz Refractor	250	,,	9 43 47·44	C. D.
Sept. 3 (*a*)	,, B.A.C. 7063	Sheepshanks Equatorial	100	,,	7 4 29·96	W.
3	,, ,,	Astrographic Equatorial	225	,,	7 4 30·52	J. S.
Oct. 30 (*b*)	,, ρ Aquarii	,, ,,	225	,,	7 10 8·60	H.
30	,, ,,	Great Equatorial	670	,,	7 10 9·15	H. F.
30	,, ,,	Sheepshanks Equatorial	100	,,	7 10 10·08	W.
30	,, ,,	Great Equatorial (Corbett)	120	,,	7 10 9·35	V.
30	Reapp. ,,	Astrographic Equatorial	225	Bright	7 46 42·54	H.
30 (*c*)	,, ,,	Sheepshanks Equatorial	100	,,	7 46 43·39	W.
Nov. 4 (*d*) (*e*)	Disapp. ξ Arietis	Astrographic Equatorial	225	Dark	6 36 18·32	H.
7 (*f*) (*g*)	,, 111 Tauri	Sheepshanks Equatorial	100	Bright	11 9(58·52)	W.
7 (*f*) (*g*)	,, ,,	Astrographic Equatorial	225	,,	11 10 (1·59)	W. S.
9 (*b*)	Reapp. λ Geminorum	,, ,,	225	Dark	9 28 1·63	H.
9	,, ,,	Old Altazimuth	100	,,	9 28 2·76	H. F.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1903.						
Nov. 9	Reapp. λ Geminorum	Sheepshanks Equatorial	100	Dark	9 28 3·06	W. S.
Dec. 2 (*d*)	Disapp. W. B. II. 1033	,, ,,	100	,,	12 18 33·44	A. C.
2 (*d*)	,, ,,	Hodgson Telescope	165	,,	12 18 34·34	P. M.
2 (*d*)	,, ,,	Astrographic Equatorial	100	,,	12 18 33·89	W. S.
4 (*h*)	,, Bradley 686	,, ,,	225	Bright	8 52 44·20	H.
4	,, ,,	Great Equatorial	670	,,	8 52 47·24	B.
4	Reapp. ,,	Astrographic Equatorial	225	,,	9 50 49·43	H.
4	,, ,,	Great Equatorial	670	,,	9 50 50·40	B.
7 (*e*)	Disapp. B. F. 1146	Astrographic Equatorial	225	,,	13 44 39·64	J. S.
7	Reapp. ,,	,, ,,	225	Dark	14 31 47·87	J.|S
10 (*i*)	Disapp. *d* Leonis	Sheepshanks Equatorial	100	Bright	13 13 50·25	W.
10	Reapp. ,,	Old Altazimuth	100	Dark	13 46 12·60	W.
31 (*b*)	Disapp. 75 Tauri	Astrographic Equatorial	225	,,	8 23 40·13	H.
31	,, ,,	Great Equatorial	670	,,	8 23 40·00	H. F.
31	,, ,,	Sheepshanks Equatorial	100	,,	8 23 39·50	W.
31 (*b*) (*j*)	Reapp. ,,	Astrographic Equatorial	225	Bright	9 6 41·34	H.
31 (*g*)	,, ,,	Sheepshanks Equatorial	100	,,	9 6(45·16)	W.

Notes.

(*a*) Not quite instantaneous. (*b*) Instantaneous.
(*c*) The observer noted "not more than 1 sec. late."
(*d*) The limb of the Moon was only slightly defective.
(*e*) The star was seen right up to the limb.
(*f*) Clouds passing. (*g*) Observation doubtful.
(*h*) The star appeared to glide up to and under the limb.
(*i*) Definition not good.
(*j*) The star appeared from a shallow indentation in the limb.

The apertures of the telescopes used are as follows :—

	inches.
Great Equatorial	28
Merz Refractor	12¾
Astrographic Equatorial (Guiding Telescope)...	10
Sheepshanks Equatorial	6¾
Great Equatorial (Corbett Telescope)... ...	6½
Hodgson Telescope	6
Old Altazimuth	4

The initals H., A. C., B., C. D., H. F., W., J. S., P. M., W. S., V., are those of Mr. Hollis, Mr. Crommelin, Mr. Bryant, Mr. Davidson, Mr. Furner, Mr. Witchell, Mr. Storey, Mr. Melotte, Mr. Stevens, and Mr. Vagg respectively.

Royal Observatory, Greenwich : 1904 *January* 5.

The "Great" Magnetic Storms, 1875 to 1903, and their association with Sun-spots, as recorded at the Royal Observatory, Greenwich. By E. Walter Maunder.

(*Communicated by the Astronomer-Royal.*)

In a paper "On the Relation between Magnetic Disturbance and the Period of Solar Spot Frequency," communicated by Mr. William Ellis to the Royal Astronomical Society in 1899, and appearing in the *Monthly Notices*, vol. lx. No. 2, pp. 142–157, Mr. Ellis has classified magnetic movement or disturbance in the following manner, Class I. including all days free from magnetic irregularity, and showing only the ordinary diurnal movement :—

Class.	Degree of Disturbance.	Greatest recorded Magnetic Movement. Declination.	Horizontal Force c.g.s. value $\times 10^5$.
I.	None	No magnetic irregularity in either element	
II.	Minor	Less than 10′	Less than 50
III.	Moderate	Greater than 10′ Less than 30′	Greater than 50 Less than 150
IV.	Active	Greater than 30′ Less than 60′	Greater than 150 Less than 300
V.	Great	Greater than 60′	Greater than 300

Adopting the same classification, there were nineteen "great" storms in the twenty-nine years from 1875 January 1 to 1903 December 31. The extreme amplitude of the movements in declination exceeded one degree in all these cases but two. In these two cases it exceeded 55′ : but as the movement in horizontal force exceeded 350 on the c.g.s. scale, they have also been ranked as "great" storms, in accordance with Mr. Ellis's definition. The nineteen are thus distinctly the most important "storms" of the entire period of twenty-nine years.

All the nineteen without exception took place when there was present on the Sun a group of spots with projected area of over 1000 millionths of the area of the visible disc ; *or* when a group, at one time very large, had returned in a diminished form to the central meridian.

With one exception—No. 18—all the storms began with an exceedingly characteristic movement of the magnets, generally only moderate in amount, but distinguished by its suddenness. This appeared simultaneously in the registers of declination, horizontal force, vertical force, and earth currents, the remarkable feature of the movement being its instantaneousness. This characteristic movement is referred to in Tables I. and II. by the word "sharp" in the column "Character of Beginning." Taking the time of this swift characteristic movement as that of the commencement of the storm, it would appear that these

nineteen storms did not begin indifferently when a great spot was anywhere on the Sun's disc, but that the most important group visible always lay within a certain region, the position of which, relative to the central meridian of the Sun, was as follows :—

				D.	H.
Extreme easterly position,	19° E.	; in time		1	11
„ westerly	47° W.	„		3	14
Mean	14° W.	„		1	2

The greatest of these nineteen storms were the following :—

Storms 4 *and* 5.—These occurred in a disturbed period lasting from 1882 April 13 to April 24 ; *i.e.* synchronous with the passage of Spot-group 729, which was visible April 13 to April 25.

Storm 4 was synchronous with a sudden outburst in Group 726 and a smaller outburst in Group 729.

Storm 5 with Group 729 at the normal position, *i.e.* one day past the central meridian.

Storm 7.—A very intense and long-continued disturbance, lasting in all from 1882 November 11 to November 26, synchronising with Group 885—the largest group of the cycle—in its entire passage across the visible disc, viz. from November 12 to November 25. The storm was the most violent of any from the beginning of 1875 up to the time of Storm 19.

Storm 9.—Synchronous with Group 2421, the greatest spot-group of the entire twenty-nine years.

Storm 10.—Synchronous with the return of the last-named group, the disturbance lasting during its entire passage across the disc—1892 March 4 to 17. A very long-continued rather than a violent disturbance.

Group 2421 at its return in 1892 March was numbered Group 2440.

Storm 14.—Disturbance lasted from 1894 February 20 to March 1. Synchronous with the passage across the disc of Group 3412, from February 16 to March 1.

Storm 19.—The greatest storm of the nineteen as regards the magnitude of the movements, but by no means the most long-continued.

The connection between the magnetic "storm" and a great sun-spot was especially clear in the cases of the first six storms noted above. The spots were amongst the most remarkable even of the "nineteen largest" groups, and the magnets were disturbed during nearly the whole time of the passage of the spots across the disc, the disturbance becoming a "great" storm near the time of central transit.

In five cases out of the 19 two considerable groups were on the disc, and it is therefore uncertain which should be associated with the storm. In the case of *Storms* 12 and 13 one of the two groups (Group 2583) was near the central meridian at the time of the beginning of both the storms. The other group (Group 2581)

seen during Storm 12 was not seen during Storm 13. Group 2581 is, therefore, probably to be associated rather with the "active" disturbance four days earlier, when it likewise had just passed the central meridian. This "active" disturbance is therefore associated with it in Table II.

Four points appear clear from the tables:—

1. There is a real connection between large Sun-spots and great magnetic storms. This is shown (*a*) by the fact that in every case a "great" storm was synchronous with the passage of a large spot or with its return; (*b*) by the correspondence of the very greatest storms to the time of visibility of the very greatest spots; (*c*) by the fact that there were no considerable storms in the interval between the two great spot-groups of 1898 September 3–15 and 1903 October 4–18. The appearance after an interval of more than five years of a "very large" group of spots was answered by a magnetic disturbance greater than any which had occurred in the interval.

2. There is a real, but only rough, connection between the size of the spot and the intensity of the storm. This is shown by the correspondence of seven out of the nineteen greatest storms to seven out of the nineteen greatest spots in a period of over twenty-nine years. The proportion of correspondences in so long a period is far too high to be due to accident. Of the nineteen spot-groups shown in Table II. the nine largest all synchronised with a "great" or "active" storm.

3. The area of the spot-group is not by any means an exact index of the degree or intensity of the magnetic disturbance. This is shown by the fact that the nineteen largest sun-spots corresponded in time to magnetic disturbances as follows:—

To "great"...	7
„ "active"	7
„ "moderate"	2
„ "minor"	2
„ "none"	1
	19

4. The "great" storms did not begin indifferently at any time of the passage of a great spot across the disc of the Sun, but during the period of five days beginning thirty-four hours before the centre of the spot-group reached the central meridian, and ending eighty-six hours after that time, the mean time being twenty-six hours after the spot reached the central line.

At the same time, as mentioned above, in some of the greatest storms a distinct disturbance began with the entrance of the group on the disc and lasted until its disappearance at the west limb. In these cases, however, a great storm of an altogether distinct character began with a typical sharp movement after the spot had reached the central region of the disc.

Details of the "storms" and spot-groups follow.

T

The "Great" Mag

The following nineteen magnetic storms comprise *all* the storms since the end of
of arc. The table therefore includes *all* the most violent storms in the last twenty-

Ref. No.	Magnetic Storm. Greenwich Civil Time. From		Period of Disturbance. To		Duration.	Character of Beginning.	Extreme Amplitude of Movement. Dec.	H.F.	V.F.	Largest Gr No. of Group.	Time of T across
		d h		d h	h		′				
1	1880 Aug.	12 12	Aug.	14 8	44	Sharp	65	·016	·008	343/5	Aug.
2	1881 Jan.	31 12	Feb.	1 3	15	„	75	18	8	412	Jan.
3	Sept.	12 12	Sept.	14 2	38	„	60	17	8	566	Sept.
4	1882 Apr.	16 23	Apr.	17 23	24	„	60+	30+	22+	726	Apr.
5		20 3		21 8	29	„	70+	20+	8	729	
6	Oct.	2 10	Oct.	3 3	17	„	60	14	No regis.	846	Sept.
7	Nov.	17 10	Nov.	21 4	90	„	110	50+	·025	885	Nov.
8	1886 Mar.	30 8	Apr.	1 3	43	„	65	20+	7	1860	Mar.
9	1892 Feb.	13 5	Feb.	14 18	37	„	70+	29+	15+	2421	Feb.
10	Mar.	11 22½	Mar.	13 5	30	„	75	26	6±	2440	Mar.
11	May	18 8	May	19 8	24	„	73	16	11	2515	May
12	July	16 12½	July	17 20	32	„	75	36+	7+	2583	July
13	Aug.	12 12	Aug.	13 2	14	„	65	34	12	2615	Aug.
14	1894 Feb.	22 22	Feb.	26 1	75	„	55	21	7+	3412	Feb.
15	July	20 6	July	21 2	20	„	60	36	14	3624	July
16	Aug.	20 2½	Aug.	20 18	15	„	65	22+	12+	3668	Aug.
17	1898 Mar.	15 1	Mar.	16 8	31	„	75	18+	10+	4702	Mar.
18	Sept.	9 14	Sept.	10 11	21	Grad.	55	20+	10	4781	Sept.
19	1903 Oct.	31 6	Nov.	1 5	23	Sharp	121	64	32	...	Oct.

Of these nineteen storms—

7 synchronised with one of the nineteen greatest spots.
1 „ „ the return of one of the nineteen greatest spots.
9 „ „ a "large" group of spots.
2 „ „ the return of a "large" group of spots.

19

A group with a projected area of $\frac{1}{1000}$th of the visible disc of the Sun is ranked as "large."

us, 1875–1903.

hich the extreme amplitude of movement in declination has amounted to one degree

n the Sun (Projected Area).

al en and it.	Date and Place on Disc when*		Mean Coord. of Group.		Area of Group on Day of Storm.		Entire Disc. Total Projected Area.	Entire Disc. Total Corrected Area.	Proportion to Mean for Year.	No. of Spots on Sun.
	First seen	Last seen	Long.	Lat.	Projected.	Corrected.				
			°	°						
1	July 20 D	Sept. 18 W	235·0	+24·5	1191	708	1850	1388	3·3	3
2	Jan. 29 D	Feb. 27 D	165·8	−13·0	1083	581	1544	1097	1·5	5
2	Sept. 6 D	Oct. 5 D	100·7	+13·6	1238	777	2332	1770	2·4	7
1	Mar. 23 D	May 10 D	93·0	−19·3	1693	880	6750	4667	4·6	8
0	Apr. 13 E	May 24 W	64·8	−28·5	3343	1908	6809	4314	4·3	7
0	Sept. 25 E	Dec. 1 W	50·2	−21·7	1647	1046	3561	2101	2·1	6
0	Oct. 20 D	Dec. 20 D	120·7	+19·2	4194	2352	4640	2771	2·7	6
4	Mar. 26 E	May 3 W	116·0	−16·8	1008	518	1641	1028	2·7	5
7	Nov. 15 E	Mar. 17 W	255·7	−28·3	5252	2999	7137	4905	4·0	6
3	Nov. 15 E	Mar. 17 W	250·3	−28·4	1111	602	1313	793	0·7	4
4	Apr. 17 D	May 22 W	84·3	−14·0	1055	589	1503	1195	1·0	5
0	June 13 D	Aug. 16 D	33·1	−31·3	1031	728	1674	1546	1·3	4
0	June 13 D	Aug. 16 D	16·5	−30·4	337	214	1394	942	0·8	7
1	Jan. 25 D	Mar. 1 W	186·9	−32·0	2263	1275	3416	1941	1·5	7
2	June 14 E	Aug. 19 W	76·6	−12·4	700	488	2790	1793	1·4	8
1	Aug. 11 E	Sept. 14 D	26·4	+ 6·4	1773	1239	2148	1466	1·1	9
4	Mar. 6 E	May 1 D	119·5	−13·2	1514	1256	1974	1559	4·1	4
3	Sept. 3 E	Nov. 6 D	239·0	−12·1	3742	2011	3759	2021	5·4	3
5	...	...	298·0	−22·0	1216	670	1737	1368	...	2

Of these nineteen storms—

5 took place with two "large" groups on the disc.
3 „ „ a "large" group east of the C.M.
11 „ „ a "large" group west of the C.M.

19

Taking the mean "Interval between Storm and Transit" for the 14 cases where there was only one "large" group, the result is $+1^{d}\ 2^{h}$, corresponding to a position of 14°·3 west of the C.M.

* D means that the spot formed or dissipated on the disc, *i.e.* in the visible hemisphere; E, that it was first seen at the east limb; W, that it was last seen at the west limb.

TABLE

The Nineteen Larges

These include *all* the groups of which the projected area exceeded on any day the area exceeded the 500th part of the visible

No. of Group.	Limiting Dates. From	To	No. of Appearance.	Total Duration of Group. Rotation.	Days.	Means for Apparition. Area.	Long.	Lat.	Day of Greatest Area. Projected. Day.	Area.	Corrected. Day.	Area.
726	1882 Apr. 10	Apr. 23	2nd	3	49	833	93°0	−19°3	Apr. 20	2517	Apr. 21	225
729	Apr. 13	Apr. 25	1st	2	56±	1744	64·8	−28·5	Apr. 18	3693	Apr. 21	210
885	Nov. 12	Nov. 25	2nd	3	62	1711	120·7	+19·2	Nov. 18	4667	Nov. 18	242
1062	1883 June 25	July 7	1st	2	53±	1353	36·2	+10·5	June 30	3573	June 30	187
1080	July 19	July 31	3rd	4	80	1321	73·9	− 9·5	July 25	3115	July 24	164
1343	1884 Mar. 31	Apr. 7	1st	4	94±	1132	19·1	−10·3	Apr. 3	2486	Apr. 6	224
1706	1885 June 15	June 27	2nd	4	106±	1363	241·0	+10·6	June 21	3767	June 21	191
2293	1891 Aug. 29	Sept. 10	4th	8	185	1212	221·5	+21·8	Sept. 5	2846	Sept. 8	179
2421	1892 Feb. 5	Feb. 18	4th	5	138±	2402	255·7	−28·3	Feb. 11	5359	Feb. 13	299
2581	July 4	July 16	2nd	5	135±	1591	83·2	+11·0	July 10	3994	July 7	238
3106	1893 Aug. 2	Aug. 13	1st	2	53±	1594	292·6	−17·6	Aug. 7	4771	Aug. 7	262
3281	Nov. 16	Nov. 28	1st	3	74±	1234	322·1	− 5·1	Nov. 20	2992	Nov. 20	168
3412	1894 Feb. 16	Mar. 1	2nd	2	43±	1208	186·9	−32·0	Feb. 21	3038	Feb. 21	174
3668	Aug. 11	Aug. 23	1st	2	42±	1246	26·4	+ 6·4	Aug. 16	3741	Aug. 16	191
3731	Oct. 2	Oct. 14	1st	1	27±	1453	59·9	−12·4	Oct. 6	4123	Oct. 5	251
4428	1896 Sept. 10	Sept. 22	1st	1	27±	1731	55·7	+13·1	Sept. 17	4254	Sept. 11	245
4497	1897 Jan. 3	Jan. 15	1st	3	81±	2132	342·8	− 6·9	Jan. 8	5239	Jan. 8	274
4781	1898 Sept. 3	Sept. 15	2nd	4	82	1556	239·0	−12·1	Sept. 10	4033	Sept. 10	223
	1903 Oct. 4	Oct. 18	1st	2	56±	1700	201·0	−22·0	Oct. 11	3800	Oct. 11	208

The figures for the spot-group of 1903 October 4–18 are estimated only.

Of the nineteen spot-groups—

7 were in the Northern Hemisphere.
12 „ „ Southern „
19

Nine spot-groups had a mean corrected area for the apparition of over 1500 millionths.

ot-groups, 1875–1903.

oth part of the visible disc of the Sun—2500 millionths, or of which the corrected misphere of the Sun—2000 millionths.

Day of Transit across C.M. Date.	Total Projected Area.	Corrected Area.	Magnetic Storm. Time of Beginning G. Civil T.	Class.	Duration.	Character of Beginning.	Extreme Amplitude of Movement. Dec.	H.F.	V.F.	Interval between Storm & Transit.
d h			d h		h		′			d h
r. 17 0	6750	4667	Apr. 16 23	Great	24	Sharp	60+	·030+	·022+	−0 1
c. 19 3	6537	3934	Apr. 20 3	„	29	„	70+	20+	8	+1 0
r. 18 20	5422	2937	Nov. 17 10	„	90	„	110	50+	25	−1 10
y 1 13	4111	2229	June 30 5	Moder.	47	Grad.	20	8	2	−1 8
y 25 21	4548	2843	July 30 0	„	84	Sharp	25	8	3	+4 3
r. 31 18	614	423	...	None	...	...	...	...	...	...
e 21 12	4483	2328	June 24 22	Active	36	Sharp	30	12	4	+3 10
t. 3 15	3432	2172	Sept. 9 10	„	70	„	33	6	5	+5 19
. 11 22	5863	3800	Feb. 13 5	Great	37	„	70+	29+	15+	+1 7
y 10 6	6391	4275	July 12 18	Active	42	„	33	22	6	+2 12
g. 7 10	8499	5128	Aug. 6 4	„	44	„	30	15	4	−1 6
. 22 8	3840	2132	...	Minor	...	...	...	...	...	...
. 22 11	4161	2707	Feb. 22 22	Great	75	Sharp	55	21	7+	+0 11
g. 17 5	3809	2288	Aug. 20 2½	„	15	„	65	22+	12+	+2 21
. 8 3	4667	2587	...	Minor	...	...	...	...	...	...
t. 16 17	4116	2021	Sept. 18 2	Active	22	Grad.	33	14	3	+1 9
. 9 10	4739	2441	Jan. 2 13	„	14	„	39	4	4	−6 21
t. 9 6	3759	2021	Sept. 9 14	Great	21	„	55	20+	10	+0 8
. 11 18	4800	2800	Oct. 12 18	Active	10	Sharp	35	...	...	+1 0

Mean +0 20

Of these 4 were synchronous with "great" storms.

„ „ 5 „ „ "active" „

9

Ten spot-groups had a mean corrected area for the apparition of over 800 millionths and less than 1500 millionths.

Of these 3 were synchronous with "great" disturbances.

„ „ 2 „ „ „ "active" „

„ „ 2 „ „ „ "moderate" „

„ „ 2 „ „ „ "minor" „

„ „ 1 „ „ „ no disturbance.

10

Storm No. 1, 1880 *August* 12–14.—Preceded by disturbance from August 11^d 12^h–August 12^d 6^h. Extreme amplitude in Dec.=23′. Followed by very quiet period.

Sun-spots on 1880 *August* $12^d{\cdot}485$.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
			°	°	°
342	321	459	324·2	+12·5	+70·4
343	795	444	234·5	+24·6	−19·3
344	211	131	225·6	−16·9	−28·2
345	396	264	213·5	+21·0	−40·3
346	127	90	216·0	−18·9	−37·8
Totals	1850	1388			

These five groups should probably be reckoned as three, since in the following rotation Nos. 343 and 345 form one almost continuous stream, and 344 and 346 another.

There were thus two important groups on the Sun at the time of the storm :—

Nos. 343–345, with total projected area 1191 and total corrected 708. This was its greatest appearance. It formed on the disc July 20, and was last seen at the west limb September 18.

Nos. 344–346, with total projected area 338, and total corrected 221. In the next rotation it was six times as large ; a very great group, with a mean corrected area for the time of appearance, September 4–16, of 1314. It was first seen, at the east limb, August 9, and last seen, at the west limb, October 14.

Storm No. 2, 1881 *January* 31–*February* 1.—Preceded by a moderate movement from January 30^d 23^h–January 31^d 7^h. Extreme amplitude in Dec.=16′. Followed by very quiet period.

Sun-spots on 1881 *January* $31^d{\cdot}452$.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
			°	°	°
411	202	360	168·8	+15·2	+22·6
412	1083	581	165·9	−12·9	+19·7
413	96	56	116·2	−16·6	−30·0
414	13	8	107·1	−14·5	−39·1
415	150	92	70·5	+18·8	−75·7
Totals	1544	1097			

Only one of these groups was seen during any other rotation.

No. 412 formed on the disc on January 29 ; passed off at the west limb, February 4 ; reappeared, east limb, on February 20 ; disappeared on the disc, February 27. January 31 was the day of its greatest area. Its rise, therefore, was very rapid.

Storm No. 3, 1881 *September* 12–14.—Preceded by two moderate movements, one from September 8^d 22^h to September 9^d 3^h; extreme amplitude in Dec.$=15'$; the other from September 9^d 21^h–September 10^d 3^h; extreme amplitude in Dec.$=14'$. Followed by an active movement, September 14^d 13^h–September 15^d 1^h; extreme amplitude in Dec.$=30'$.

Sun-spots on 1881 *September* $13^d{\cdot}583$.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
			°	°	°
566	1238	777	101·6	+13·7	+49·4
568	342	216	88·4	− 3·3	+36·2
569	23	13	27·4	+17·2	−24·8
570	51	30	21·5	+19·6	−30·7
571	228	224	351·9	+17·0	−60·3
572	421	473	348·0	+22·1	−64·2
573	29	37	344·2	+17·8	−68·0
Totals	2332	1770			

Of these groups none were seen in the previous rotation, and only two in the following, viz. Nos. 566 and 572. No. 566 formed on the disc, September 6, and disappeared likewise on the disc, October 5. No. 572 was not nearly so large a group as No. 566, and was first seen, September 12, at the east limb, disappearing on the disc, October 14.

Storms Nos. 4 and 5, 1882 *April* 17 and *April* 20–21. These two storms occurred in a disturbed period. Its history is as follows :—

April 13^d 23^h–14^d 7^h. Fluctuations in Dec. ($\pm 3'$), with wave at commencement ($-10'$), and wave at 14^d 4^h ($+10'$).

April 14^d 12^h–15^d 1^h. Fluctuations in Dec. ($\pm 3'$) with wave at $18\frac{1}{4}^h$ ($-8'$).

April 15^d 23^h–16^d 4^h. Fluctuations in Dec. ($\pm 3'$).

April 16^d $13\frac{1}{2}^h$–19^h. Fluctuations in H.F. ($\pm{\cdot}0015$).

April 16^d $23\frac{1}{2}^h$. Storm No. 4 begins.

April 17^d 23^h. Storm No. 4 ends.

April 18^d $20\frac{1}{4}^h$. Sharp wave in Dec. ($-6'$).

April 20^d 3^h. Storm No. 5 begins. Oscillations sometimes 14 or 15 an hour.

April 20^d 12^h. Very rapid oscillations end.

April 21^d 8^h. Storm No. 5 ends.

April 21^d 22^h. Wave in Dec. ($+8'$).

April 23^d $20\frac{1}{2}^h$–24^d 9^h. Fluctuations in Dec. ($\pm 5'$).

Sun-spots on 1882 *April* $17^{d}\cdot587$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
722	115	285	160·3	+21·1	+75·2
728	970	940	144·0	−17·8	+58·9
726	1693	880	92·5	−18·3	+ 7·4
732	35	19	72·2	−21·9	−12·9
729	3388	1963	65·3	−28·7	−19·8
733	317	177	60·4	−13·8	−24·7
734	160	200	20·6	+10·9	−64·5
735	72	203	4·4	−11·6	−80·7
Totals	6750	4667			

Sun-spots on 1882 *April* $21^{d}\cdot409$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
731	28	70	113·9	−18·7	+79·3
726	2402	2258	92·3	−17·5	+57·7
729	3396	2109	64·0	−28·7	+29·4
733	221	125	62·2	−13·2	+27·6
734	444	237	22·7	+11·4	−11·9
735	339	193	6·6	−10·2	−28·0
738	68	124	321·3	+ 7·1	−73·3
Totals	6898	5116			

Only three of these groups were considerable.

No. 726 was seen in the preceding and also in the following rotation. It formed and dissipated in the visible hemisphere, being first seen March 23, and last seen May 10. It appeared at the east limb April 10, and underwent a sudden and great development on April 17, increasing from an area of 156 on April 16 to an area of 1371 on April 18. This was synchronous with Storm No. 4.

No. 729 came on at the east limb on April 13 with the first slight fluctuations of the magnetic needle, and passed the C.M. on April 19. It passed off at the west limb on April 25, returned in the next rotation, and was last seen at the west limb, May 24.

No. 728 developed from an area of 769 on April 16 to an area of 940 on April 17, and diminished afterwards. This increase seems to have been coincident with the much greater increase of No. 726. It was first seen, April 11, on the disc, and last seen, May 5, also on the disc.

All three groups were at their greatest development during the present apparition.

Storm No. 6, 1882 *October* 2–3.—Preceded by a quiet time. Followed by fluctuations, October 4^d 16^h–23^h, in Dec. ($\pm 5'$), and by an "active" disturbance, October 5^d 18^h–October 6^d 16^h (Dec. $\pm 16'$). Fluctuations in Dec. ($\pm 5'$) from October 8^d $19\frac{1}{2}^h$ to October 9^d 1^h and October 9^d $19\frac{1}{2}^h$ to October 10^d 2^h. October 10^d $18\frac{1}{4}^h$–$20\frac{1}{2}^h$; wave in Dec. ($-20'$).

Sun-spots on 1882 *October* $2^d \cdot 560$.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
845	16	11	66°·3	+21°·0	+42°·1
846	1647	1046	50·0	−21·7	+25·8
847	119	64	40·4	+20·3	+16·2
848	911	534	34·6	−22·9	+10·4
849	823	418	32·7	+ 8·0	+ 8·5
850	45	28	348·3	+ 6·6	−35·9
Totals	3561	2101			

Three of the groups were considerable, and two of them, Nos. 846 and 848, were in the same latitude, and only about 12° apart in longitude. These, therefore, probably formed only a single disturbance. In the following rotation the two groups appear to have coalesced. This double group, 846+848, was by far the most important visible on October 2. It was seen during three rotations, being first seen, September 25, at the east limb, and last seen, December 1, at the west limb. It attained its greatest area during the present apparition.

Storm No. 7, 1882 *November* 17–21.—A very long-continued disturbance, beginning about November 11^d 21^h and lasting until November 26^d 22^h. Characteristic sharp swings occurred November 14^d 14^h, November 15^d 8^h, November 16^d 8^h, November 17^d 10^h, November 19^d 13^h, November 25^d $16\frac{1}{2}^h$. The great storm lasted November 17^d 10^h–November 21^d 4^h; but from November 11^d 21^h–November 13^d 23^h was an "active" disturbance, of extreme amplitude $50'$ in Dec. Two other "active" disturbances followed, the one November 21^d 16^h–24^h; amplitude $40'$ in Dec.; the other November 25^d $16\frac{1}{2}^h$–November 26^d 3^h; amplitude $37'$.

Sun-spots on 1882 *November* $17^d \cdot 277$.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Long. from C.M.
877	42	100	216°·9	−24°·4	+75°·6
887	56	35	173·3	+19·2	+32·0
884	86	44	148·1	− 7·8	+ 6·8
885	4194	2352	120·1	+19·1	−21·2
888	124	104	88·5	− 6·3	−52·8
889	137	136	81·5	+10·9	−59·8
Totals	4639	2771			

There was only one considerable group, and that one, No. 885, was the largest in the whole cycle, 1878 to 1889, as Storm No. 7 was the most violent magnetic storm in the same period.

This group formed on the disc, October 20 ; passed off at the west limb, October 28 ; returned to the east limb, November 12 ; passed off at the west limb, November 25 ; returned to the east limb, December 10 ; disappeared on the disc, December 20. Its mean area during the apparition November 12–25, the period of its greatest development, was 1711.

Storm No. 8, 1886 March 30 to April 1.—Preceding the storm there was a double wave in Dec. (+8′ to −8′), March 29^d $0\frac{3}{4}^h$–$3\frac{1}{2}^h$. There were also fluctuations of about ±3′ in Dec. every day from March 15 to March 28.

The storm was followed by fluctuations in Dec. (±4′), April 1^d 6^h–11^h, and a wave (−10′) $18\frac{1}{2}^h$–19^h.

The storm itself was divided into two short but sharp disturbances, the one March 30^d 8^h–March 31^d 5^h, the other March 31^d 11^h–April 1^d 3^h. In both the oscillations attained a frequency of 20 per hour.

Sun-spots on 1886 *March* $30^d{\cdot}232$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
1856	57	172	200·2	+ 5·5	+76·3
1859	525	273	136·5	− 1·2	+12·6
1860	1008	518	115·3	−16·8	− 8·6
1861	34	47	53·7	−11·9	−70·2
1862	14	18	56·1	− 8·7	−67·8
Totals	1638	1028			

Three of these groups were seen in other rotations, viz. Nos. 1856, 1859, and 1860. Of these, Group No. 1860 was the most considerable. It was first seen, March 26, at the east limb, and last seen, May 3, at the west limb. Its mean area during the present apparition, in which it attained its greatest development, was 487.

Storm No. 9, 1892 February 13–14.—Preceded by small daily fluctuations (±3′), with occasional waves of 7′ to 10′ in Dec. from January 28 to February 12.

Followed by a moderate disturbance (17′) February 15^d 13^h–24^h ; and by small daily fluctuations until February 20^d 18^h–February 21^d 4^h, when another moderate disturbance took place (24′). This began with a characteristic sharp movement.

Sun-spots on 1892 *February* 13^{d}·408.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
2421	5252	2999	255°·4	−28°·6	+19°·6
2425	1023	529	239·5	−19·4	+ 3·7
2426	134	100	193·2	+15·9	−42·6
2427	203	191	180·2	+ 9·6	−55·6
2428	273	374	169·5	+12·8	−66·3
2429	253	712	158·4	+15·4	−77·4
Totals	7138	4905			

Of these six groups No. 2421 was much the largest. It was first seen on the east limb on 1891 November 15. The present appearance was therefore during its fourth rotation. It appeared on the east limb on February 5, passed off at the west limb on February 18, and its mean area for the apparition was 2402, the largest group in the eleven-year cycle 1879–1901.

Storm No. 10, 1892 March 11–13.—A long-continued but not very intense disturbance preceded the storm. It commenced February 29^{d} 21^{h} and continued until March 9^{d} 5^{h}. Extreme amplitude 45′ in Dec.

After the storm there were fluctuations in Dec. (±3′) March 15^{d} 20^{h}–March 16^{d} 3^{h}, with wave (−15′) March 15^{d} 22^{h}–16^{d} 0$\frac{1}{2}^{h}$.

Sun-spots on 1892 *March* 11^{d}·152.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
2443	68	36	260°·6	−15°·7	+17°·0
2440	1111	602	250·5	−28·5	+ 6·9
2442	103	110	181·1	−23·4	−62·5
2444	31	45	173·0	−26·0	−70·6
Totals	1313	793			

The only important group was No. 2440, the return of group No. 2421, the premier group of the cycle. No. 2440 was seen on the east limb, March 4, and reached the west limb, March 17. Its mean area was 386. It did not return.

Storm No. 11, 1892 May 18–19.—Preceded by moderate fluctuations and waves from May 16^{d} 22^{h}–May 17^{d} 24^{h}. Extreme amplitude in Dec. 20′. Followed by small sharp movements from May 19^{d} 15^{h}–May 20^{d} 8^{h} (±3′).

Sun-spots on 1892 *May* $18^{d}\cdot439$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
2515	1055	589	$84^{\circ}\cdot4$	$-14^{\circ}\cdot2$	$+22^{\circ}\cdot6$
2519	284	149	$50^{\circ}\cdot4$	$+10^{\circ}\cdot0$	$-11^{\circ}\cdot4$
2522	14	7	$41^{\circ}\cdot9$	$+6^{\circ}\cdot2$	$-19^{\circ}\cdot9$
2523	4	4	$7^{\circ}\cdot1$	$-25^{\circ}\cdot5$	$-54^{\circ}\cdot7$
2524	147	446	$342^{\circ}\cdot1$	$+14^{\circ}\cdot9$	$+79^{\circ}\cdot7$
Totals	1504	1195			

Three of these were considerable groups.

Group No. 2515 was the largest of the three, with a mean area of 742 for the rotation, and had passed the central meridian about a day and a half at the time of the storm. No. 2524 was actually on the east limb at the time. No. 2519, the smallest of the three, was one day short of reaching the central meridian.

No. 2515 formed on the disc, April 17; made its second appearance, May 10, at the east limb; and disappeared, May 22, at the west limb. No. 2519 appeared first at the east limb, May 14, and disappeared on the disc, June 17. No. 2524 formed on the disc, April 23, and was last seen, June 26, at the west limb.

Storm No. 12, 1892 *July* 16–17.—Preceded by "active" disturbances: one from July $12^{d}\ 18^{h}$–July $13^{h}\ 12^{d}$, the other from July $13^{d}\ 18^{h}$–July $14^{d}\ 11^{h}$. Followed by small rapid fluctuations in Dec. July $21^{d}\ 0^{h}$–8^{h}, with wave $2\frac{1}{2}^{h}$–$4\frac{1}{2}^{h}$ ($+20'$).

Sun-spots on 1892 *July* $16^{d}\cdot240$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
2581	177	393	$81^{\circ}\cdot1$	$+10^{\circ}\cdot4$	$+77^{\circ}\cdot5$
2583	1031	728	$31^{\circ}\cdot7$	$-31^{\circ}\cdot3$	$+28^{\circ}\cdot1$
2584	183	108	$25^{\circ}\cdot7$	$+28^{\circ}\cdot4$	$+22^{\circ}\cdot1$
2586	283	317	$301^{\circ}\cdot6$	$-9^{\circ}\cdot5$	$-62^{\circ}\cdot0$
Totals	1674	1546			

All four groups were considerable, and all were seen during more than one rotation.

Of the four groups No. 2581 was much the largest and longest lived, but it was passing off at the west limb when the storm began. It attained its greatest area, 1591, during this apparition.

No. 2583 was the second in size, and was at its largest during this apparition. Mean area 829.

It seems doubtful which group should be associated with the spot, probably No. 2583. The groups were first seen:—No. 2581, June 7, east limb; No. 2583, June 13, on disc; No. 2584, June 12, on disc; and No. 2586, July 15, east limb. They were

last seen :—No. 2581, Oct. 5, west limb ; No. 2583, August 16, on disc ; No. 2584, July 20, west limb ; and No. 2586, August 22, west limb.

Storm No. 13, 1892 *August* 12–13.—Preceded by two "moderate" disturbances, viz. August 3^d 14^h–23^h, and August 4^d $14\frac{1}{2}^h$–August 5^d 1^h. A quiet period followed the storm.

Sun-spots on 1892 *Aug.* $12^d{\cdot}280$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
2615	337	214	15·4	−30·5	+ 9·4
2620	144	73	11·4	+14·5	+ 5·4
2621	212	112	13·7	+25·0	+ 7·7
2617	127	66	11·3	− 8·4	+ 5·3
2618	106	57	354·0	−14·2	−12·0
2622	367	299	313·3	+10·4	−52·7
2623	104	121	302·8	− 7·4	−63·2
Totals	1397	942			

None of the groups were large, but two were returns of groups seen on the disc during Storm No. 12, viz. No. 2615, a return of No. 2583 ; and No. 2623, a return of No. 2586. This is important, as it renders it probable that the other two groups seen during Storm No. 12, viz. Nos. 2581 and 2584, were not connected with that disturbance. The group No. 2615, the return of No. 2583, appears the most likely group on this occasion. Its mean area during the rotation when it attained its greatest development was 829 millionths, July 7–19 ; whilst No. 2622, the next most important group, was only 426 during the rotation Sept. 7–18.

Storm No. 14, 1894 *February* 22–26.—Preceded by "active" disturbance, amplitude 36′ in Dec., beginning with characteristic sharp movement at February 20^d $20\frac{1}{2}^h$ and lasting till February 22^d 6^h. Followed by "active" disturbance, amplitude 50′ in Dec., beginning with characteristic sharp movement at February 28^d 15^h, and lasting till March 1^d 3^h. Minor fluctuations followed till March 2^d 20^h.

Sun-spots on 1894 *February* $23^d{\cdot}210$.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
3408	94	53	200·3	−26·4	+21·8
3418	2	1	193·5	−27·7	+15·0
3412	2263	1275	187·5	−31·8	+ 9·0
3420	241	123	174·5	−15·7	− 4·0
3413	366	214	152·7	−25·8	−25·8
3416	155	90	151·0	+ 3·7	−27·5
3417	295	185	142·1	−16·1	−36·4
tals	3416	1941			

Group No. 3412 was the only large one. It formed on the disc, January 25, passed off at the west limb, January 29; reappeared at the east limb, February 16; and passed off at the west limb, March 1. It did not return again. Mean area during the present rotation 1208, its greatest development.

Storm No. 15, 1894 *July* 20–21. Preceded and followed merely by the small fluctuations common throughout the year.

Sun-spots on 1894 *July* 20^{d}·176.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Long. from C.M.
			°	°	°
3624	700	488	77·0	−12·5	+40·5
3628	25	14	64·6	−11·5	+28·1
3629	559	299	57·3	+ 6·9	+20·8
3630	542	298	53·7	−11·2	+17·2
3635	14	7	44·5	−16·6	+ 8·0
3637	542	351	357·3	+12·8	−39·2
3638	403	326	344·7	+14·7	−51·8
3640	5	10	320·5	−10·1	−76·0
Totals	2790	1793			

Groups No. 3624 and No. 3637 were the largest and longest lived, both having passed their greatest area during an earlier rotation. Of the two No. 3624 was not only the larger now, but was much the larger at greatest development, and was now in an earlier phase of its decline. It was first seen, June 14, at the east limb, and last seen, August 19, at the west limb. It attained its greatest area during the previous apparition, 1894 June 14–25.

Storm No. 16, 1894 *August* 20.—Preceded and followed merely by the small fluctuations common throughout the year.

Sun-spots on 1894 *August* 20^{d}·398.

No. of Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Longitude from C.M.
			°	°	°
3681	5	5	41·7	+ 5·2	+58·1
3668	1773	1239	28·4	+ 6·9	+44·8
3682	6	4	19·1	−17·6	+35·5
3683	17	8	349·8	+ 7·1	+ 6·2
3672	24	13	346·9	−19·3	+ 3·3
3674	162	83	336·3	− 5·8	− 7·3
3675	25	17	321·0	−26·7	−22·6
3678	29	21	299·7	−12·5	−43·9
3679	106	76	297·7	+17·8	−45·9
Totals	2147	1466			

Only three of these were considerable or long-lived—viz. Nos. 3668, 3674, and 3679; and of these Group No. 3668 was by far the most important. It was first seen, August 11, at the east limb, and last seen, September 14, on the disc. It attained its greatest area during the present apparition, 1894 August 11–23.

Storm No. 17, 1898 *March* 15–16.—Preceded by an "active" disturbance, March 11^d 14^h–March 12^d 6^h; amplitude in Dec. = 30′. The movements, however, were slow and not numerous.

The maximum range of movement in the storm itself was over 60′, thus ranking it as a "great" storm; but the storm was not a violent one in other characteristics, the oscillations being comparatively few and slow. Only the fluctuations ordinary to the year followed.

Sun-spots on 1898 *March* $15^d{\cdot}372$.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Longitude from C.M.
			°	°	°
4702	1514	1256	122·7	−13·3	+53·2
4703	131	102	116·3	+11·0	+46·8
4704	322	197	104·4	− 8·2	+34·9
4706	7	4	94·2	− 3·7	+24·7
Totals	1974	1559			

Of these groups No. 4702 was much the largest. No. 4706 was very small and short-lived, and the other two groups were only about one-fourth or one-fifth the area of No. 4702, even when they were largest. Group No. 4702 was first seen, March 6, at the east limb, and last seen, May 1, on the disc. It attained its greatest area during the present apparition, 1898 March 6–17.

Storm No. 18, 1898 *September* 9–10.—Preceded by two successive waves in Dec. (−7′ and −6′) on September 8^d 16^h–20^h.

The storm scarcely amounted to a "great" one. The extreme amplitude in Dec. was not quite a degree, and the oscillations were not very frequent.

Followed by movements lasting until September 11^d 12^h, about 18′ in Dec.

Sun-spots on 1898 *September* $9^d{\cdot}624$.

No. of Group.	Total Area. Projected.	Total Area. Corrected.	Heliographic Longitude.	Heliographic Latitude.	Longitude from C.M.
			°	°	°
4782	10	6	248·4	− 7·2	+15·3
4781	3742	2011	238·1	−11·7	+ 5·0
4783	7	4	230·9	−18·4	− 2·2
Totals	3759	2021			

The only important group was No. 4781, the other two being very small and seen only on the one day.

No. 4781 was seen during three rotations—viz. September 3–15, September 30–October 12, and October 28–November 6. Its first appearance was at the east limb on September 3 ; its last appearance, on the disc, on November 6. It attained its greatest area during the present apparition, 1898 September 3–15.

Storm No. 19, 1903 *October* 31–November 1.—Preceded by a wave in Dec. and H.F. at October $30^{d}\ 21^{h}$, and then by small movements up to the time of the characteristic instantaneous movement.

Sun-spots on 1903 *October* $31^{d}\cdot426$.

Letter for Group.	Total Area. Projected.	Corrected.	Heliographic Longitude.	Latitude.	Longitude from C.M.
			°	°	°
a.	1216	670	296·0	−20·0	− 5·8
b.	520	698	232·0	+17·5	−69·8
Totals	1736	1368			

Group *a* appeared at the east limb on October 25, and reached the west limb November 6. The measures for October 31 are approximate only ; not definitive.

Further Note on the "Great" Magnetic Storms, 1875–1903, *and their association with Sun-spots.* By E. Walter Maunder.

The facts brought out in the preceding paper are, so far as they go, in entire agreement with the facts tabulated in the important paper by the Rev. W. Sidgreaves read before the Society 1900 December 14. The difficulty is in their interpretation. It has been urged that if there be a real direct connexion between a particular great sun-spot and a great magnetic storm, then that connexion should always show itself. Every great spot on the Sun should be answered by a great magnetic storm here ; every great magnetic storm should be synchronous with the appearance of some great spot ; and, further, the violence of the storm should bear a definite relation to the extent of the spot. This relation does indeed hold good if we consider only the average of a number of instances ; it does not hold good universally in every individual case.

An example is in the immediate recollection of everyone. The sun-spot which crossed the central meridian on 1903 October 12 was one of the six or seven largest groups seen during thirty years. The magnetic storm which occurred at this

time was an "active" one, not a "great" one. The sun-spot of October 31 was a large one, but not of extraordinary dimensions—less than one-third the size of that of October 12. Synchronous with its passage over the central meridian was the greatest magnetic storm that has been recorded for thirty years—perhaps the greatest that has yet been recorded at Greenwich. The natural expectation would have been that the reverse should have been the case, that the greater spot should have corresponded to the greater storm, and the smaller to the feebler one.

Yet such an expectation, however natural, involves a number of assumptions, for some of which we have very little warrant. It assumes, for example, that the efficiency of a spot is greatest at the moment of its attaining its greatest area. More important still, it assumes that its influence is equally great in all directions; that there is nothing like *direction* in the forces or emanations which proceed from that disturbed region of the Sun of which the spot is the visible indication. Yet this assumption is founded upon no sufficient evidence, even if it be not a wholly gratuitous one.

If we consider the solar corona we recognise at once that it is not symmetrically distributed round the Sun, evenly thinning out to exactly the same extent in all directions. On the contrary it is highly structured. Whatever conception we form of its nature, we are obliged to consider the lines which compose it as essentially lines of force: they indicate regions where action is greater than in the neighbouring darker areas. The eclipses of the last eight years have been specially instructive on this point. The corona of 1896, as photographed both by Sir George Baden-Powell's expedition to Nova Zembla and by MM. Kostinsky and Hansky of the Russian expedition, showed one great lobe whose outlines curved together till they united to form one long straight ray. In the eclipse of 1898 no fewer than four such synclinal regions were seen, one of which was triple, all of which terminated in straight rays of prodigious length. The longest was actually photographed for a distance from the Sun's centre which could not have been less than five millions of miles. In the two following eclipses, 1900 and 1901, the same features were perceived, though the long rays were not photographed to the same distance as in 1898; and there is no reason to doubt that they are features of every eclipse. That these extensions were not photographed before 1896 is no doubt due simply to the circumstance that a sufficient exposure and extent of field was not employed in order to secure them.

If we suppose that the effect of a solar disturbance travels outwards in somewhat the same manner as these long coronal rays—in other words, that the effect is greatest in some one direction which need not be truly radial, since the great coronal rays are not necessarily so—this will remove the ground of the difficulty before us. The intensity of any magnetic storm due to a solar disturbance would then depend upon two factors:

first, the actual magnitude of the disturbance itself, and next, upon the distance of the Earth from the direction of maximum effect. We should find, as we actually do, that when the average was taken of a large number of cases the frequency of magnetic storms and their intensity would correspond to the size of the solar spots; but at the same time we should also find, as we do, that there would be a wide margin of irregularity in special instances. It is in perfect accord with this suggestion that we actually find at the moment of commencement of the nineteen great storms examined that the most important spot on the Sun was always found within a restricted area on the surface. If the influence of the spot were exactly equal over the whole sphere of which it was the centre it is difficult to understand why this relation should have shown itself.

In the foregoing remarks I have confined myself entirely to the spots. We have at present no sufficient material for a similar discussion in the case of faculæ, prominences, or flocculi. In the ordinary way we see prominences only round the limb, faculæ only near it; of flocculi we have not yet enough observations; spots, on the other hand, we see wherever they exist in any part of the hemisphere turned towards us, and our knowledge of them may be said to be fairly complete. Further, the four different orders of phenomena are not independent, but interdependent; and concerning the first three we know that they go through their variations in the course of a solar cycle in substantial accord. At the present time, whatever may be the case in the future, spots are the most easily observed and most fully observed of all the various phenomena which can afford us any index of the solar activity.

Suggested Connection between Sun-spot Activity and the Secular Change in Magnetic Declination. By Mrs. Walter Maunder.

(*Communicated by E. Walter Maunder.*)

A connexion between the Sun and terrestrial magnetism has been recognised as existing in the following relations:—

(1) Daily.—The diurnal range of magnetic declination, dip, and intensity, according to the hours of local mean time, when by means of the Earth's rotation different parts of its surface are exposed to the action of the Sun's rays.

(2) Yearly.—The annual variation in the amount of the diurnal range corresponding to the variation in the presentation of any particular locality to the Sun's rays in the course of the year.

(3) Cyclical: "Eleven-year" Period.—Variation in the

amount of the diurnal range synchronous with the variation of the spotted area of the Sun.

In terrestrial magnetism this cycle is shown in the variation of the diurnal range, both of magnetic variation, dip, and intensity, and is evidenced, moreover, both (*a*) by the frequency of storms and (*b*) by the variation of the diurnal range when cleared of storms.*

On the Sun the "eleven-year" cycle is shown by sun-spots, faculæ, prominences, and corona, and in the case of sun-spots is evidenced both (*a*) by the frequency of giant spots alone and (*b*) by the variation in the spotted area of the Sun when cleared of giant spots.

These three periodic variations in the Earth's magnetism, which are thus known to vary in sympathy with the Sun, form, according to Mr. L. A. Bauer, but a small part of the whole magnetic force of the Earth—less than 5 per cent.—and they are generally ascribed to electric currents in the upper regions of the atmosphere. At least 95 per cent. of the Earth's magnetism is to be referred to causes within the crust, largely to a system of electric currents imbedded deep within the interior of the Earth. In particular the secular change in the three elements of terrestrial magnetism (declination, dip, and intensity) should probably be referred largely to the effect of secondary electric currents generated within the Earth by its rotation round an axis not coincident with its magnetic axis.

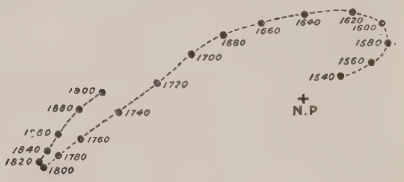
It being thus the case that the secular change in the Earth's magnetism seems primarily to be due to internal causes, it is all the more interesting to note that certain critical changes in the Earth's magnetism appear to have synchronised with certain critical changes in the activity of the Sun.

The following tables show the secular change in the magnetic declination at London (long. 0°., lat. $51\frac{1}{2}$°) for every twenty years from 1540 to 1900, and for Baltimore (long. $76\frac{1}{2}$° W., lat. $39\frac{1}{2}$° N.) from 1640 to 1900:—

Year.	London.	Baltimore.	Year.	London.	Baltimore.
1540	7°·2 E.	...	1740	15°·3 W.	3°·2 W.
1560	9·6 E.	...	1760	19·6 W.	2·0 W.
1580	10·9 E.	...	1780	22·7 W.	1·0 W
1600	10·1 E.	...	1800	24·1 W.	0·7 W.
1620	7·3 E.	...	1820	24·1 W.	0·9 W.
1640	3·3 E.	5·3 W.	1840	23·2 W.	1·8 W.
1660	0·6 W.	6·0 W.	1860	21·6 W.	3·0 W.
1680	3·9 W.	6·1 W.	1880	18·7 W.	4·3 W.
1700	7·1 W.	5·5 W.	1900	16·5 W.	5·4 W.
1720	11·0 W.	4·5 W.			

* "On the Relation between Magnetic Disturbance and the Period of Solar Spot Frequency," by Mr. W. Ellis, F.R.S., *Monthly Notices*, vol. lx. pp. 142–157.

In the table for London it will be noted that there is a maximum declination E. about 1580, zero declination about 1660, maximum declination W. about 1810, since which date it has been returning eastward. For Baltimore there was a maximum declination W. about 1680, a minimum declination about 1800, since which date it has been again increasing westward. The diagram (fig. 1) will perhaps show more clearly the path of the point of intersection of the two magnetic meridians. The curve for the period between 1540 and 1640 is drawn



+ Baltimore

+ London

FIG. 1.

on the supposition that the needle at Baltimore varied then in the same manner as it did during the period of observation.

The path of the magnetic pole, as this intersection of the magnetic meridian may perhaps be called, would seem not to be a mere revolution round the geographical pole, but to be more like the swing of a pendulum. Evidently there are nodes or points where the direction of its motion changed about 1580; between the years 1640 and 1680, when it was moving rapidly and was at the same time near the north pole of the Earth; and about 1800.

When other pairs of stations in Europe and America are

PROF. WOLFER'S "RELATIVE SUN-SPOT NUMBERS" SMOOTHED FOR THE "ELEVEN-YEAR" PERIOD.

taken, the curves traced by the intersection of their magnetic meridians is of the same character as that shown by the two selected.

Sun-spots were first discovered in the year 1610, and since that period we know of two striking cases of prolonged solar quiescence. The first was the most pronounced, and lasted from about the year 1640 to about the end of the century, during which period but a single spot now and then appeared even at times when the Sun should have been most active. The "eleven-year" cycle seemed obliterated, and an almost unbroken solar calm prevailed. Auroræ were notedly absent.

The second period of solar quiescence was remarkable, but not so complete. The "eleven-year" cycle could still be plainly traced, but for several succeeding maxima the solar activity was much below the normal; and this was emphasised by the great activity that preceded and followed the period of semi-quiescence. The accompanying diagram, which gives Wolfer's relative spot numbers cleared of the "eleven-year" cycle, will show this plainly (Plate 13). During this period of solar quiescence auroræ were also deficient.

We have, of course, no means of saying what was the state of the Sun's activity earlier than 1610. At that period spots were abundant, but the stationary position eastward of the magnetic declination needle in the years 1580–1600 may also have been marked by a period of quiescence in solar activity and in auroræ.

Prior to 1600 there seems to have been a change in the law of secular change. Mr. L. A. Bauer (in his "Magnetic Tables and Isogonic Charts for 1902") has found both for Rome and for Fayal Island, in the Azores, that there is a marked difference for the fifteenth and sixteenth centuries between the curve as given by the present apparent law of secular change and the full curve as deduced from observations obtained by the aid of early "compass charts."

There is no material for discussing the secular change of any of the magnetic elements in the southern hemisphere, neither are there observations of dip or magnetic intensity for a sufficient length of time in the northern; and these are necessary for a full discussion of the question. As it is, I have been obliged to assume for London and Baltimore that they are uninfluenced by local magnetism, or at least that such local magnetism does not affect the law of secular change. Of this there is no evidence for or against.

To sum up. The ordinary sun-spot cycle has twice been disturbed within the period of observation by long-continued calms. The first and most remarkable took place in the latter half of the seventeenth century; the second at the commencement of the nineteenth. These two periods of solar calm appear to have been answered on the Earth by corresponding periods of absence of auroræ. There was thus sympathy between solar

quiescence and terrestrial magnetism. When the movement of the north "magnetic pole" is considered it appears that at these two very epochs it was passing through two critical points of its path—the first when it was at its nearest approach to the geographical pole, the second when it was at its greatest elongation from it.

The Aurora and Magnetic Disturbance.
By William Ellis, F.R.S.

In a paper that appeared in the *Monthly Notices* of the Society for 1899 December, on the relation between magnetic disturbance and the period of solar-spot frequency, I showed from the observations of the fifty years 1848 to 1897 at the Royal Observatory, Greenwich, the general relation existing between the period of solar-spot frequency and the frequency of magnetic disturbance in the progression from Sun-spot maximum to Sun-spot minimum, and again from Sun spot minimum to Sun-spot maximum ; and further pointed out that, in addition, there existed in the frequency of magnetic disturbance an annual inequality that has no counterpart in the march of Sun-spot frequency. In a following paper appearing in the *Monthly Notices* for 1901 June, I compared this seasonal variation in the frequency of magnetic disturbance at Greenwich with the variation in frequency of the *Aurora* in the same locality, showing that in both phenomena there existed maximum epochs at or near the equinoxes, and minimum epochs at or near the solstices. I now desire to pursue the question a little further.

As regards the *Aurora*, although in our latitude there is, as mentioned, maximum of frequency at the equinoxes and minimum of frequency at the solstices, this condition undergoes modification in higher latitudes, the winter minimum becoming less pronounced as higher latitudes are approached, until it altogether disappears. In the recently published *Catalog der in Norwegen bis Juni* 1878 *beobachteten Nordlichter*, J. Fr. Schroeter has combined the work of Tromholt for Norway and Rubenson for Sweden, and formed abstracts that give, separately for each one of five latitudinal regions over Scandinavia, the monthly frequency of the *Aurora* as found from observations, many thousands in number, made during the years 1761 to 1877. And from a paper by Mr. R. C. Mossman "On the Aurora Borealis in London," contained in the *Journal of the Scottish Meteorological Society*, third series, vol. xi. p. 58, corresponding information is to be found for other positions south of the above. To these I have added the monthly frequency of magnetic disturbance at Greenwich and Paris. The various numerical results are given in the annexed table, in each case in percentage of the total frequency.

Month.	Frequency of the Aurora. Scandinavia, 1761–1877. Latitude North.					North East of Scotland, 122 years.	Edinburgh, 81 yrs.	London, 189 years.	Frequency of Magnetic Disturbance. Royal Observatory, Greenwich, 50 years.	Parc St.-Maur, Paris, 5 years.
	North of 68½°.	68½° to 65°.	65° to 61½°.	61½° to 58°.	South of 58°.	57½°.	56°.	51½°.	51½°.	49°.
July	0·0	0·0	0·0	0·4	0·6	0·4	1·8	1·9	7·0	7·9
Aug.	0·4	1·1	2·8	5·7	4·9	4·4	6·4	5·6	7·5	10·7
Sept.	7·8	9·7	13·1	13·6	14·9	12·9	12·3	14·5	9·9	12·1
Oct.	15·1	14·6	14·2	13·8	13·5	15·8	13·9	16·9	10·4	9·3
Nov.	14·4	14·0	12·8	10·4	10·3	12·0	11·7	9·6	8·4	8·7
Dec.	15·7	14·1	11·5	9·6	8·2	9·6	4·8	6·4	7·1	5·6
Jan.	16·4	15·3	13·2	9·5	8·2	10·9	10·0	8·6	8·0	5·9
Feb.	13·8	14·6	12·3	11·2	11·9	12·7	12·3	10·5	9·9	6·7
Mar.	14·8	13·7	14·5	13·5	12·6	12·0	13·9	10·2	10·3	10·2
Apr.	1·6	2·9	5·4	10·9	13·3	7·1	9·0	10·7	8·9	7·7
May	0·0	0·0	0·2	1·3	1·5	2·2	3·6	4·0	7·1	8·2
June	0·0	0·0	0·0	0·1	0·1	0·0	0·3	1·1	5·5	7·0

The numbers for the five Scandinavian regions are taken from page 420 of the before-mentioned *Catalog*, and those for the North-east of Scotland, Edinburgh, and London from the paper by Mr. Mossman. These are all percentage values. As regards magnetic disturbance at Greenwich, the days of moderate, active, and great disturbance, appearing in Table I. of my paper (*Monthly Notices*, 1899 December), have been combined and re-arranged for each month respectively on the aggregate of the fifty years employed. The days thus included are those on which disturbance occurred in declination greater than 10′, or in horizontal force greater than ·00050 c.g.s. The resulting values (monthly number of days) commencing with July are 282, 301, 401, 420, 339, 285, 322, 400, 416, 359, 289, and 223, which were converted into percentage values for insertion in the table. For Paris, M. Moureaux, in a paper *Sur la périodicité des perturbations de l'aiguille aimantée horizontale à l'Observatoire du Parc Saint-Maur*, has given, for the five years 1883 to 1887 * the number of disturbances exceeding in declination 3′ and in horizontal force ·00020 c.g.s. for each month on the aggregate of the whole five years. But as the variation in the monthly values is very similar in both elements, I have for the present purpose combined them in one series, giving for the several months commencing with July the numbers 622, 846, 955, 729, 687, 444, 468, 527, 801, 607, 643, and 554 respectively, which, similarly

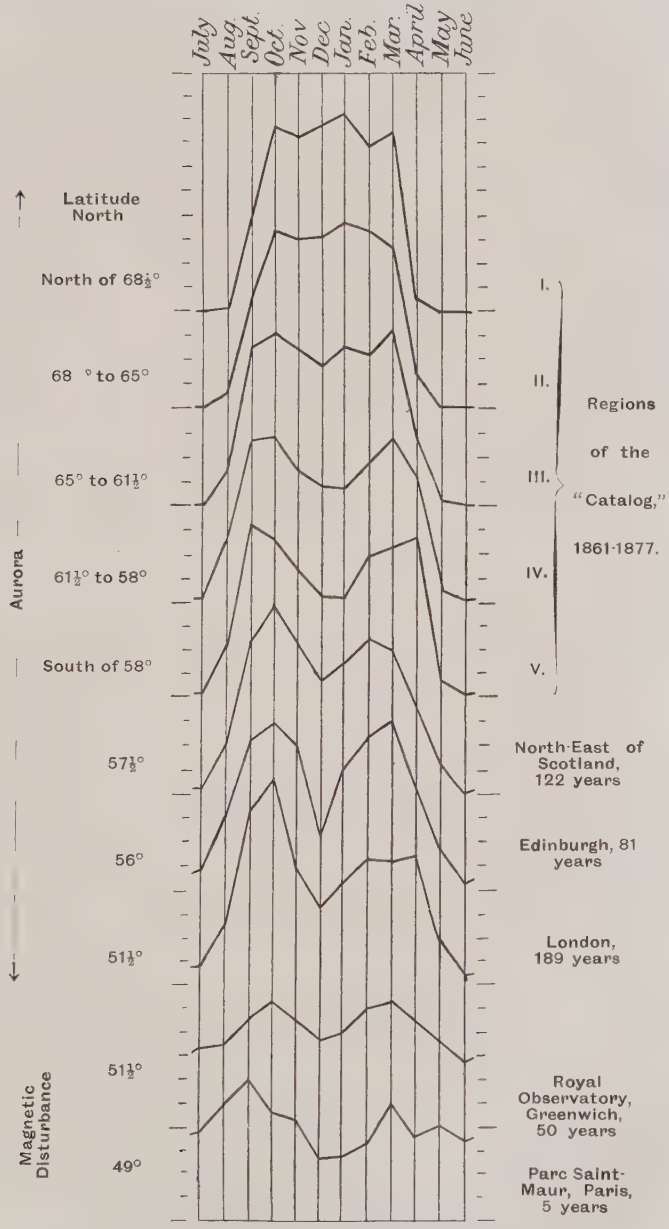
* *Annales du Bureau Central Météorologique de France*, Année 1887. *Mémoires*, p. B. 35. (In a later volume for 1897 M. Moureaux has given similar information for the fifteen years 1883 to 1897.)

converted into percentage values, are the numbers appearing in the table, the contents of which are graphically represented in fig. 1 (Plate 14).

It will be remarked that the curves of frequency of magnetic disturbance at Greenwich and Paris are very similar, showing maxima at or near the equinoxes and minima at or near the solstices, and similar also to that of the aurora at London, excepting that the autumn maximum of the latter is of more pronounced character than that of spring, as is also the case in the north-east of Scotland. It will be further observed that the strongly marked winter minimum of the aurora in lower latitudes becomes less and less marked as more northern latitudes are approached; in regions IV. and III. it is distinctly less marked, and in regions II. and I. it disappears. The autumn maximum visibly tends to become later, and the spring maximum somewhat earlier as the higher latitudes are approached, the winter depression at the same time diminishing until eventually there remains only a mid-winter maximum at or near the winter solstice. The mid-winter minimum of lower latitudes being thus converted in higher latitudes into a mid-winter maximum, it becomes a question, remarking the similarity of the auroral and magnetic curves in lower latitudes, as to what happens in higher latitudes as regards magnetic disturbance. Does it still run with the aurora? That is, does the winter depression of frequency in higher latitudes become similarly converted into a mid-winter maximum. By analogy this might be expected to be the case. But is it so? The summer effect is not easily discussed as regards aurora, because of the daylight towards the north; the winter effect is, however, readily followed, and the condition of things is clear, but information appears to be wanting as regards what happens in the case of magnetic disturbance.

It may be well to give some further indication of the really persistent character of the seasonal variation in frequency of magnetic disturbance in our latitude. The agreement between the curves for Greenwich and Paris shown in fig. 1 is sufficient evidence of the existence of the inequality, but the question may be somewhat further illustrated. For this purpose the number of days of magnetic disturbance at Greenwich during four years about each of the Sun-spot maxima of 1848, 1860, 1870, 1884, and 1894 were grouped together as indicating periods of greater frequency of disturbance, and similarly for four years about each of the Sun-spot minima of 1856, 1867, 1879, and 1890, as indicating periods of lesser frequency of disturbance, the two groups so formed containing twenty and sixteen years respectively, as follows, the comparision showing here, in addition to the law of frequency, also the absolute number of days involved.

Fig. 1.—Annual Inequality of Aurora and of Magnetic Disturbance.



Each division of the scale represents 2·0 of the percentage values of the Table. The longer lines of the scale indicate the zeros of the several curves.

Number of days in the twenty years of greater frequency of magnetic disturbance (near Sun-spot maximum).

	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
1848–1851	22	13	32	36	25	24	25	35	22	21	22	13
1858–1861	24	33	38	44	14	29	22	25	44	33	20	24
1869–1872	32	41	45	39	31	26	30	39	50	54	34	27
1882–1885	28	27	29	29	35	20	20	32	30	34	23	24
1892–1895	41	22	48	49	36	32	35	53	56	30	32	36
Total for 20 years ...	147	136	192	197	141	131	132	184	202	172	131	124
Monthly Mean ...	7·4	6·8	9·6	9·9	7·0	6·5	6·6	9·2	10·1	8·6	6·5	6·2

Yearly average = 94·4 days.

Number of days in the sixteen years of lesser frequency of magnetic disturbance (mean Sun-spot minimum).

	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
1854–1857	9	13	21	18	16	11	17	30	25	24	14	6
1865–1868	20	26	43	50	30	6	25	41	30	24	18	12
1876–1879	4	3	7	4	12	8	7	4	7	5	9	3
1887–1890	20	27	25	29	35	17	23	37	22	22	22	10
Total for 16 years ...	53	69	96	101	93	42	72	112	84	75	63	31
Monthly Mean ...	3·3	4·3	6·0	6·3	5·8	2·6	4·5	7·0	5·3	4·7	3·9	1·9

Yearly average = 55·6 days.

The monthly means of the two groups are graphically represented in fig. 2. We see for the years about maximum of Sun-spots that, on the average, magnetic disturbance occurred on some ninety-four days of the year, and about minimum of Sun-spots on some fifty-five days. But the point to which attention is to be drawn (see fig. 2) is the circumstance that the annual inequality is shown not only in periods of greater frequency of magnetic disturbance near to Sun-spot maximum, but that it is also strongly marked in periods of lesser frequency near Sun-spot minimum. It will be seen also that with some few irregularities the inequality can be very well followed in the separate four yearly groups of years about Sun-spot maximum and Sun-spot minimum. It may be added that in the fourteen years of the fifty years employed that remain intermediate between the epochs of maximum and minimum and of minimum and maximum of Sun-spots, the winter minimum is much less pronounced, the summer minimum remaining much the same, on the whole producing a flatter curve, as indicated by the thin line in fig. 2, for which (without giving all the corresponding figures) it may be added that the monthly means (number of days) commencing with July are 5·9, 6·9, 8·1, 8·7, 7·5, 8·0, 8·4, 7·4, 9·3, 8·0, 6·8, and 4·9. The annual inequality is thus most strongly developed near epochs of maximum and minimum of Sun-spots.

It may be that the flatter intermediate curve is in some measure due to accidental circumstances.

The magnetic disturbance that has been here dealt with is, as before mentioned, that taken as including days of moderate, active, and great disturbance at Greenwich, according to the convention of the previously mentioned paper of 1899 December. But it may be further pointed out that considering separately the frequency of these three different degrees of disturbance, it is found that the excess of the equinoctial frequency over the solstitial frequency is greater, the greater the degree of disturbance. The three degrees are thus defined :—(1) moderate, indicating disturbance in declination greater than 10′ but less than 30′, and in

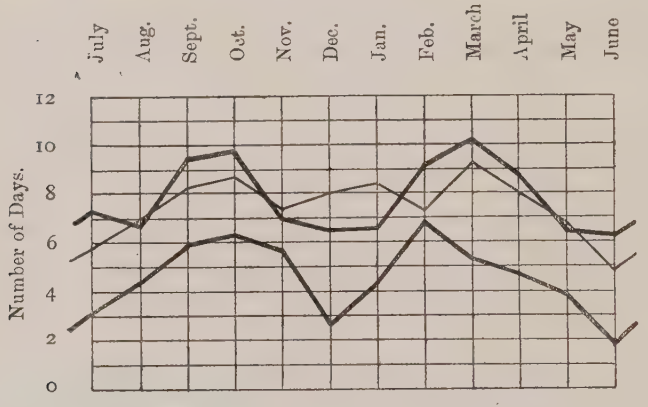


FIG. 2.—Annual Inequality of Magnetic Disturbance at Greenwich.

The upper thick line indicates the annual inequality in frequency of disturbance near maximum of Sun-spots, and the lower thick line that near minimum of Sun-spots ; the thin line represents the inequality in years intermediate between these extreme epochs.

horizontal force greater than ·00050 but less than ·00150 c.g.s. ; (2) active, indicating disturbance in declination greater than 30′ but less than 60′, and in horizontal force greater than ·00150 but less than ·00300 c.g.s. ; and (3) great, indicating disturbance in declination greater than 60′, and in horizontal force greater than ·00300 c.g.s., the three months of February to April, and from August to October, being taken as equinoctial, and the remaining six months as solstitial.

Degree of Disturbance.	Number of Days.	Frequency of Disturbance (Percentage). Equinoctial.	Solstitial.	Excess of Former.
Moderate	3,663	56	44	12
Active	299	61	39	22
Great	75	71	29	42

In this table no regard is paid to Sun-spot frequency, the whole fifty years being alike employed. It thus finally appears :

(1) That the annual inequality in frequency of magnetic disturbance at Greenwich is strongly marked, not only at periods of greater frequency and correspondingly greater magnitude of disturbance near Sun-spot maximum, but also at periods of lesser frequency and correspondingly lesser magnitude of disturbance near Sun-spot minimum.

(2) Also that the annual inequality in frequency of disturbance is greater as the degree of disturbance becomes increased.

These conclusions may appear to be in some sense contradictory. But it is to be remarked (fig. 2) that although the lower thick line includes a less number of days—that is, on the whole, a lesser numerical frequency of disturbance—than does the upper thick line, the inequality in frequency is of similar character in both. For although the frequency of active and great disturbance influences mainly the upper thick line, it also affects in no inconsiderable degree some of the years that become included in the formation of the lower thick line.

Now, if disturbance frequency progresses as does the $11\pm$ year Sun-spot frequency—that is, on the whole, with no annual period,* the mean monthly number of days of disturbance taken through a series of years should similarly, as the number of years employed is increased, more and more approximate to equality of value in all months of the year, by gradual elimination of the transient fluctuations of value, instead of which the combination of years really brings out a definite annual inequality in frequency to be observed, as already mentioned, in the four yearly groups of years, but distinctly shown in the aggregate, alike in the years of maximum frequency and minimum frequency of disturbance, diminished in the latter case as to the absolute number of days of disturbance in the various months of the year, but still showing a most marked annual inequality, which can only be understood as an undoubted physical effect.

The annual inequality in the frequency of magnetic disturbance in our latitude has thus a real existence. Our knowledge of the relations subsisting between solar and other phenomena, so far as our own geographical position is concerned, has also reached a certain definite stage—that is to say, as regards the $11\pm$ year Sun-spot cycle, a period of great solar activity is also one of great magnetic activity with frequent displays of the aurora,† and a period of solar quiet is similarly also one of magnetic quiet, with no exhibitions of the aurora. It is thus apparent :—

(1) That in addition to the progression in frequency of magnetic disturbance and of the aurora in harmony with the $11\pm$ year Sun-spot period, there exists in both phenomena, in

* *Monthly Notices*, 1899 December, p. 152.

† At Greenwich magnetic disturbance is not necessarily accompanied by aurora, but when aurora does appear there is also magnetic disturbance.

our latitude, an annual inequality having maxima at or near the equinoxes and minima at or near the solstices to which there is no counterpart in the progression of solar-spot frequency.*

(2) That the equinoctial maxima of frequency in the case of the aurora disappear in higher latitudes, becoming merged into a single midwinter maximum, with entire disappearance of the winter minimum.

Thus, viewing the relation that exists in our latitude between the annual inequality of magnetic disturbance and of the aurora, and the modification that the inequality undergoes in higher latitudes in the case of the aurora, it becomes of interest to ask whether in the case of magnetic disturbance any similar modification of the inequality occurs? As already remarked, I do not know of any available information bearing thereon.

To turn now from the question of the annual inequality in frequency of the aurora to another matter, the relation of the aurora to the $11\pm$ year Sun-spot period, it appears from observations made at Ivigtut, in Greenland,† in latitude 61° north, at Godthaab in 64° north, and at Jacobshavn in 69° north, that years of greater frequency of the aurora were years of lesser frequency of Sun-spots. But Rubenson ‡ having discussed the Swedish observations made between the years 1721 and 1877, has represented them by formulæ, n being the number of years after the initial epoch, as follows :—

$$1728{\cdot}2 + 11{\cdot}07 \times n, \text{ for maximum,}$$
$$1734{\cdot}2 + 11{\cdot}03 \times n, \text{ for minimum,}$$

and found that the observed epochs of maximum and minimum frequency of the aurora during the period mentioned were, with a few exceptions, in close agreement with corresponding epochs calculated from the formulæ, indicating also a near agreement with the corresponding maximum and minimum epochs of Sun-spots. Rubenson suspects that there may be long period variations in the aurora, but as to the existence of such variations in these and related phenomena there appears at present, I think, to be a good deal of uncertainty. But as to the $11\pm$ year period his results appear to be opposed to those found at stations in Greenland as above given, which indicate greater frequency of aurora with lesser frequency of Sun-spots, although situated in the same latitudinal zone. It may be of interest to examine a little more closely this point. In Rubenson's work (p. 298) a table is given showing the epochs of maximum and minimum frequency of the aurora as determined from the Swedish observa-

* The annual inequality in the diurnal range of the magnetic elements is similarly a terrestrial effect that has no counterpart in the progression of Sun-spot frequency.

† *Exploration Internationale des Régions Arctiques*, 1882–3, *Expédition Danoise*, tome i. p. 19.

‡ *Catalogue des Aurores Boréales observées en Suède depuis le xvi^me^ siècle jusqu'à l'année* 1877.

tions, those epochs which are supposed to be more or less doubtful being placed in parentheses. Omitting these latter, and comparing the remaining values with the corresponding Sun-spot epochs taken from Professor Wolfer's paper in the *Meteorologische Zeitschrift* for 1902 May, we have the annexed table.

Observed Epochs of Maximum and Minimum Frequency of the Aurora and of Sun-spots Compared.

Maximum Epochs			Minimum Epochs.		
Rubenson. (Aurora.)	Wolfer. (Sun-spot.)	Excess of Former.	Rubenson. (Aurora.)	Wolfer. (Sun-spot.)	Excess of Former.
1729·9	1727·5	+2·4	1736·6	1734·0	+2·6
1741·2	1738·7	+2·5	1744·3	1745·0	−0·7
1749·9	1750·3	−0·4	1756·0	1755·2	+0·8
1761·0	1761·5	−0·5	1777·8	1775·5	+2·3
1788·3	1788·1	+0·2	1798·6	1798·3	+0·3
1804·6	1805·2	−0·6	1811·8	1810·6	+1·2
1819·6	1816·4	+3·2	1824·4	1823·3	+1·1
1831·7	1829·9	+1·8	1834·2	1833·9	+0·3
1839·0	1837·2	+1·8	1844·7	1843·5	+1·2
1851·0	1848·1	+2·9	1856·3	1856·0	+0·3
1871·3	1870·6	+0·7			
Mean		+1·3	Mean		+0·9

The epochs of maximum and minimum frequency of the aurora thus appear to fall respectively at or near to maximum and minimum epochs of Sun-spots as in our latitude, but following the Sun-spot epochs a little later in time, and on the whole by a longer interval at maximum epoch than at minimum epoch. There are irregularities, however, in the progression of auroral frequency as well as in that of Sun-spot frequency. Thus the 1788·3 observed maximum epoch of aurora was a greatly accelerated one, arriving 6·3 years earlier than the calculated time by the formula for maximum above given, which is 1794·6. And Professor Wolfer, who gives the observed Sun-spot maximum as 1788·1, has also added formulæ to represent the observed values, as follows :

$$1749{\cdot}37 + 11{\cdot}091 \times n, \text{ for maximum,}$$
$$1744{\cdot}21 + 11{\cdot}141 \times n, \text{ for minimum,}$$

from which the calculated time of maximum is found to be 1793·7, the observed time being thus 5·6 years earlier. That acceleration of the auroral maximum by 6·3 years should be accompanied by an acceleration of the Sun-spot maximum by 5·6 years is a striking proof of the interrelation of the two phenomena, since they thus remain in close accord, the auroral maximum exceeding the Sun-spot maximum (see the preceding table)

by only 0·2 year. We may also compare the formulæ of Rubenson for aurora with those of Wolfer for Sun-spots by reducing them to two epochs, one near the beginning and one near the end of the series for aurora employed, as follows :—

Maximum Epochs.			Minimum Epochs.		
of Aurora. (Rubenson.)	of Sun-spots. (Wolfer.)	Excess of Former.	of Aurora. (Rubenson.)	of Sun-spots. (Wolfer.)	Excess of Former.
1750·3	1749·4	+0·9	1745·2	1744·2	+1·0
1872·1	1871·4	+0·7	1866·6	1866·8	−0·2

giving mean differences of +0·8 year and +0·4 year respectively as compared with +1·3 year and +0·9 year, found by comparison of the epochs as observed, in which latter (as before mentioned) a few epochs of aurora understood to be somewhat doubtful were not included. These results indicate that throughout Sweden, during the period covered by the observations discussed, the times of maximum and minimum frequency of the aurora were, on the whole, in close relation with those of maximum and minimum frequency of Sun-spots, both phenomena being, with the 11± year periodical variation of magnetic diurnal range and of magnetic disturbance, all alike the effect of some common unknown cause.

What is already known as regards the relations existing between solar-spots, terrestrial magnetism, and the aurora in middle latitudes may be but an outer fringe of the subject, but corresponding information in regard to polar regions is not to the same extent available. As regards magnetism, observations in those regions have hitherto been more or less of a temporary nature. But to obtain any sufficient knowledge of the physical constitution of the globe, observatories, not of temporary but of permanent character, are necessary in all regions, as well as in the more habitable portions of the globe, and the north polar regions offer opportunities for placing such observatories as do not exist in the same way in the south.

Our Earth is more and more found to be not an isolated body simply receiving light and heat from the Sun, as formerly commonly conceived, but one subject to other external influence made manifest to us in various ways with which we have only in later times for the most part become acquainted, but of the precise nature of which little more is known than that, complex in action, it seems to be one that may pervade the whole solar system, involving in one common field of action all the bodies thereof. If we could plant observatories on some of the other planets of our system, and communicate therewith, how enlarged might be our knowledge of the action of the forces that surround us.

Note on Mr. Ritchey's Photographs of the Andromeda Nebula.

By W. H. Wesley.

(*Communicated by the Secretaries.*)

At the meeting of the Society in 1903 March there were exhibited a series of photographs of nebulæ taken by Mr. G. W. Ritchey with the 24-inch reflector of the Yerkes Observatory. The photographs were remarkable as showing in the *Andromeda* and *Orion* nebulæ the detail both in the bright central portions and in the faint outlying parts, and the question was raised as to whether some selective process of development or printing had been employed.

In the *Monthly Notices* for 1903 May (vol. lxiii. p. 395) appeared a letter from Mr. Ritchey fully explaining the process he adopted in developing astronomical negatives. Mr. Ritchey further says that "in the case of such objects as the *Orion* and *Andromeda* nebulæ something more is needed if the detail in the bright central parts and the faint extensions are both to be shown on the same positive. One negative is reserved precisely as developed by the above method. For transparencies and lantern-slides a second negative is exposed in the telescope, developed as above described, and the parts which are so dense that they cannot be printed are reduced locally by the use of a very weak reducing solution. Much time and care are given to this, and an attempt is made to keep the relative brightness of the various parts the same in kind, though not in degree, as in the untouched negative, which is constantly used for comparison during the process."

The two beautiful transparencies, therefore, which Mr. Ritchey presented to the Society, and which were shown at the March meeting, were taken from negatives which had been locally reduced. The advantages of showing on the same plate the detail of all parts of the nebula are sufficiently obvious. At the same time, while the utmost care was evidently taken with the reduction, it seemed very desirable to compare these photographs with entirely untouched negatives to find if any false effects or spurious detail had been introduced by the process.

A short time ago Mr. Ritchey sent, through Mr. Hinks, two untouched negatives of the *Andromeda* nebula, so that, with regard to this nebula the opportunity was afforded for such a comparison. The negative from which the enlarged transparency was made had an exposure of four and a half hours ; and as one of the untouched negatives had been exposed four hours it was particularly suitable for the comparison. The detail of the nebula is so complex that I principally concentrated my attention upon the finer detail immediately surrounding the great central condensation, as this was most liable to be affected by the local reduction. I examined with special care the edges of

the dark rifts, where the bright bands are broken up into very minute flocculent patches. The result of my examination is that I find nothing on the transparency that is not on the negative. I specially looked for any *edge* of greater brightness which might mark the limit which the reducing solution reached, but can find nothing of the kind. I can therefore only conclude that the process of local reduction has been so carefully carried out that no spurious detail has been produced in the negative from which this transparency was made.

Since the prolonged examination of small details renders one extremely liable to overlook larger and more general effects, I have repeatedly broken off my examination and resumed it on other occasions ; but the other results have always been the same. Mr. Hinks and I have also together compared the photographs, but we both came to the conclusion that whatever was in the transparency could be found in the untouched negatives.

New Double Stars detected with the $17\frac{1}{4}$ *in. Reflector during the year* 1903. By T. E. Espin, M.A.

The following stars have been found to be double during the year 1903. As they are so few, and the measures are for the most part incomplete, I have not numbered them.

B.D.	R.A. 1880	Decl.	P.	D.	Mags.
°	h m	° ′	°	″	
53·234	1 2·5	+53 59	215·7	5·1	8·6 11·8
63·1346	17 21·4	63 51	19·1	6·5	9·0 11·5
64·1256	18 15·6	64 1	332·7	8·6	8·2 12·0
51·2372	20·9	51 35	198·7	2·7	8·6 8·7
60·1844	42·4	60 32	103·6	4·3	9·1 11·1
61·1816	19 5·2	61 4	243·9	6·1	9·1 9·8
59·1979	12·7	59 33	116·3	7·4	9·0 11·6
59·1981	13·3	59 34	113·7	8·1	8·8 11·7
64·1346	20·2	64 18	216·3	4·4	8·8 9·9
64·1364	35·6	64 47	19·3	8·9	8·5 10·5
64·1369	37·8	64 39	313·7	2·7	8·8 9·4
64·1386	46·3	64 23	70·7	6·5	8·0 10·5
59·2160	57·3	59 25	145·0	4·1	9·0 11·5
52·2883	21 8·7	52 48	—	4 ±	9·0 12
63·1814	22 5·8	63 31	—	4 ±	9 11

The Rotation Period of Saturn in 1903. By W. F. Denning.

The extensive disturbance observed on Saturn in 1903 offered an excellent opportunity for determining the rotation period of the north-temperate region of the planet. No doubt, however, the rate of the spots merely represented that of an atmospheric current, not nearly conformable with the motion of the globe.

To ascertain the rotation period reliably and within small limits of error it is by no means necessary to depend upon the transit times of spots derived by micrometric measurement. The method of eye estimation may not be quite so exact, but, as in the case of *Jupiter's* markings, it is capable of furnishing excellent results. There are certainly more important factors affecting such investigations than the manner of taking transits. The question of *identification* is a serious one. The mis-identification of planetary features has proved a fruitful source of large errors, and is scarcely avoidable unless observations are obtained at short intervals and a fairly numerous list of transits accumulated.

Large irregular markings, light and dark, were pretty abundant in the northern hemisphere of *Saturn* during the past year, and these underwent certain variations in aspect. The form, size, and brilliancy of the luminous spots appeared notably inconstant, and there were changes in the rate of motion. Several of the objects alluded to were compound, consisting of two or three parts wholly or partly divided by dusky masses or wisps similarly to those sometimes seen crossing the bright equatorial region of *Jupiter*. To follow the same individual marks on *Saturn*, and safely single them out one from another after various intervals, formed the most critical and delicate work in the recent telescopic study of the planet. As a test of the correctness of my own identifications I have collected together all the observed transits of spots I could find by various observers, and the comparisons made have, I hope, been the means of eliminating serious mistakes. I here append the transit times from observations secured between June 23 and September 22 of Barnard's spot, and of two smaller spots following it marked B, C, and D respectively. The times are compared with an adopted, uncorrected rate of $10^h\ 38^m$ and the residuals given.

Observed Transit Times of Barnard's White Spot on Saturn and of two other White Spots following it. 1903, *June 23–September 22.*

Observer.	Date. 1903.	Transit G.M.T. h m	Computed Period $10^h\ 38^m$. h m	Residual O−C. h m	Spot.	Notes.
Barnard ...	June 23	21 42	21 42	± 0	B	b., e., o.
Barnard ...	24	18 58	18 58	± 0	B	b., e., o.
Hartwig ...	26	13 36	13 30	+ 6	B	
Messow and Mainka	26	13 38	13 30	+ 8	B	

Observer.	Date. 1903.	Transit G.M.T. h m	Computed Period $10^h\ 38^m$. h m	Residual O−C. h m	Spot.	Notes.
Sola...	June 26	13 19	13 30	— 11	B	d.
Hough	27	21 18·7	21 24	— 5·3	B	m.m.
Wilson	July 1	21 18	21 6	+ 12	B	b., e.
Graff	4	13 19	12 54	+ 25	B or C?	
Hough	6	18 49·4	18 4	+ 45·4	C	m.m.
Sola...	8	12 40	12 36	+ 4	B	
Sola...	9	10 0	9 52	+ 8	B	
Sola...	11	14 45	15 2	— 17	B	
Fauth	11	15 10	15 2	+ 8	B	
Brenner	12	12 18	12 18	± 0	B	e., m.m.
D. ...	12	12 50	12 18	+ 32	C	
Phillips	12	12 30	12 18	+ 12	B	
Kibbler	12	12 30	12 18	+ 12	B	
Hough	13	20 15·3	20 12	+ 3·3	B	m.m.
Barnard	13	20 11	20 12	— 1	B	b., e., s. spot f.
Barnard	14	17 26	17 28	— 2	B	s. spot f.
D. ...	16	11 52	12 0	— 8	B	v. b., l., e.
Brenner	16	12 0	12 0	± 0	B	m.m.
Brenner	17	9 16	9 16	± 0	B	m.m.
Hough	18	17 39·7	17 10	+ 29·7	C	m.m.
Sola...	20	11 32	11 42	— 10	B	
Sola...	21	9 45	8 58	+ 47	C	
Burnham	22	18 6	16 52	+1 14	D	
Sola...	24	11 25	11 24	+ 1	B	
D. ...	24	11 17	11 24	— 7	B	l. b.
Kibbler	24	11 30	11 24	+ 6	B	d.
Williams	24	11 34	11 24	+ 10	B	d.
D. ...	24	12 40	11 24	+1 16	B	s. o.
Sola...	28	11 15	11 6	+ 9	B	
Burnham	29	19 20·9	19 0	+ 20·9	B	orC? m.m.
Burnham	30	17 26·5	16 16	+1 10·5	D	m.m.
Williams	31	13 21	13 32	— 11	B	o.
Barnard	Aug. 2	19 57	18 42	+1 15	D	s., b., o., e
Phillips	4	13 5	13 14	— 9	B	
D. ...	5	10 20 10 31	10 30	— 10 + 1	B	b., e.
Brenner	5	10 30	10 30	± 0	B	m.m.
Phillips	5	10 40	10 30	+ 10	B	

Observer.	Date. 1903.	Transit G.M.T. h m	Computed Period $10^h\ 38^m$. h m	Residual O−C. h m	Spot.	Notes.
D.	Aug. 5	11 35	10 30	+1 5	D	
D.	12	12 24	12 38	− 14	B	b., σ.
Williams ...	13	10 14	9 54	+ 20	B	s.
Brenner ...	17	9 36	9 36	± 0	B	
Burnham ...	19	15 31·2	14 46	+ 45·2	C	f.
Brenner ...	21	9 18	9 18	± 0	B	s., m.m.
D.	21	10 9	9 18	+ 51	C	r. s., o.
D.	25	9 12	9 0	+ 12	B	
D.	25	10 18	9 0	+1 18	D	
D.	29	8 37	8 42	− 5	B	b., o.
D.	29	10 3	8 42	+1 21	D	
D.	Sept. 2	8 28	8 24	+ 4	B	
D.	2	9 36	8 24	+1 12	D	
D.	5	10 44	10 50	− 6	B	o.
Williams ...	5	11 5	10 50	+ 15	B	s.
D.	6	7 50	8 6	− 16	B	f.
D.	6	8 40	8 6	+ 34	C	
Williams ...	9	10 41	10 32	+ 9	B	b.
Williams ...	13	10 24	10 14	+ 10	B	d.
D.	14	8 8	7 30	+ 38	C	
Williams ...	17	10 7	9 56	+ 11	B	s., b., o.
D.	17	10 9	9 56	+ 13	B	o.
D.	18	7 25	7 12	+ 13	B	
D.	22	6 49	6 54	− 5	B	o.
D.	22	8 12	6 54	+1 18	D	

In first column transits by D. = Denning.

In last column: b., bright; f., faint; d., double; e., extended; l., large; s., small; o., obvious; m.m., micrometric measure; v., very; r., rather; p., preceding; f., following.

The discussion of a considerable number of transits of bright spots recorded here and elsewhere show that the corrected rotation period was as nearly as possible

$$10^h\ 38^m\ 3^s$$

between the latter part of June and middle of September. Then a marked acceleration apparently occurred, and from the observations made at Bristol, July to December, of fifteen bright and dark spots the mean rate of rotation was about

$$10^h\ 37^m\ 56^s.$$

I shall be glad if Professor Hough will look into the foregoing table of transits, for I am bound to conclude that in his interesting paper in *Monthly Notices* for 1903 December, p. 122, he has incorrectly identified the very few (though doubtless accurate) observations taken by himself at the Dearborn Observatory and by Professor Burnham at the Yerkes Observatory. The rotation periods he has deduced are therefore too great. The observation of August 19, $15^h\ 31^m{\cdot}2$ G.M.T., was certainly not of Barnard's spot at all, but of a smaller object further south, and following it about three-quarters of an hour. Professor Barnard (*Astronomical Journal*, No. 547, p. 180) specially alludes to a smaller spot, following on July 13 and 14, the principal one which he was the first to discover. The same object also came under observation on several occasions at Bristol. Professor Hough, in his paper alluded to, also identifies spots seen on July 6, 22, and 30 (period $10^h\ 38^m\ 30^s{\cdot}5$), but the first of these was different, and very probably the same as that of August 19, marked C in the foregoing table. The transits of July 22 and 30 undoubtedly referred to the same marking (marked D in the table), and this object was well observed by Barnard on August 2, and many times at Bristol in July and subsequent months, following Barnard's spot about an hour and a quarter.

I entirely differ from Professor Hough in his depreciation of eye-estimated transits of markings on *Jupiter* and *Saturn*. Professor Hall, in his valuable and accurate observations of the white equatorial spot which he discovered on *Saturn* in 1876 December, relied upon eye estimations, and concluded the probable error of determining the time when the spot was at the centre of the disc to be ± 3 min. on a good night and ± 5 min. on a bad one. Professor Barnard's two pairs of eye-estimated transits of his recent spot clearly prove two things—viz. that its period was just about $10^h\ 38^m$, and that the four observations were very accurate and comparable with the best instrumental measures :—

Date.	Transit Time.	Rate of $10^h\ 38^m$	Difference.
	h m	h m	min.
June 23	21 42	21 42	±0
24	18 58	18 58	±0
July 13	20 11	20 12	−1
14	17 26	17 28	−2

In the table giving the combined results of observers the residuals are certainly rather large in some cases. This is due to several causes—viz. the bad definition often prevailing, the changing aspects of the spots, and their duple or triple appearance in several instances. And it is quite possible that the individual markings, like the equatorial spots on *Jupiter*, presented irregularities of motion, causing them to oscillate to and fro about their mean positions.

It will be noticed that Herr Brenner's six observations of Barnard's spot agree to the very minute with the uncorrected rotation period of $10^h\ 38^m$. I cannot explain this singular circumstance.

With reference to the transits of minor spots obtained by Professors Hough and Burnham, I may add that I made eight observations of each of Burnham's spots of July 23, $17^h\ 28^m$ and $17^h\ 57^m$ G.M.T. Hough's transit of August 20, $15^h\ 32^m{\cdot}3$ G.M.T. seems to be of the former marking of the pair. It followed Barnard's spot about $3^h\ 24^m = 115°$. I have twelve observations of Burnham's spot of August 9, $17^h\ 32^m{\cdot}6$ G.M.T., which preceded Barnard's spot about $3^h\ 10^m = 107°$.

Observations of *Saturn* were made here on seventy-eight nights between July 1 and December 11. The last good observation of Barnard's spot was on November 24, $4^h\ 17^m$, when it preceded the zero meridian (rate $10^h\ 38^m$ uncorrected) 27 minutes. The spot C in the table I appear to have lost after September, but D I retained in view until December 10, when it was in transit at $4^h\ 30^m$, and followed the z.m. 52 minutes.

This z.m. requires to be corrected chiefly for differences in the longitude and distance of *Saturn*. Between the end of June and middle of September (the period comprised in the table) there would be a *plus* correction of more than 2 seconds per rotation for the relative displacement in longitude. Between the end of June and beginning of December there was a slight *minus* correction of about 1 second, the far greater distance of *Saturn* more than compensating for the variation in longitude.

I hope to place before a later meeting of the Society some further details of my observational results, with a determination of the mean rotation period.

Bishopston, Bristol: 1904 *January* 4.

Ephemeris for Physical Observations of

Greenwich Noon. 1904.		P.	L−O.	B.	Apparent Diameter. Equat. Diam.	Excess over Polar.	Defect of Illum.	*d*.	Q.
		°	°	°	″	″	″	°	°
May	1	335·580	239·076	+2·442	34·22	2·20	0·07	5·12	245·01
	5	335·693	239·986	2·474	34·39	2·21	0·08	5·67	245·33
	9	335·808	240·883	2·506	34·57	2·22	0·10	6·20	245·63
	13	335·927	241·768	2·538	34·77	2·23	0·12	6·71	245·91
	17	336·048	242·635	2·569	34·99	2·25	0·14	7·21	246·18
	21	336·172	243·485	2·600	35·23	2·26	0·16	7·70	246·43
	25	336·300	244·317	2·630	35·48	2·28	0·18	8·16	246·66
	29	336·428	245·128	2·661	35·75	2·30	0·20	8·61	246·88
June	2	336·558	245·918	2·692	36·05	2·32	0·22	9·03	247·10
	6	336·688	246·686	2·726	36·36	2·34	0·24	9·43	247·31
	10	336·817	247·429	2·758	36·69	2·36	0·26	9·81	247·52
	14	336·945	248·145	2·789	37·04	2·38	0·28	10·16	247·71
	18	337·072	248·834	2·819	37·41	2·41	0·31	10·48	247·90
	22	337·196	249·492	2·845	37·80	2·43	0·33	10·77	248·06
	26	337·318	250·121	2·870	38·20	2·46	0·35	11·03	248·22
	30	337·436	250·716	2·895	38·63	2·49	0·37	11·26	248·37
July	4	337·548	251·277	2·920	39·07	2·52	0·39	11·45	248·51
	8	337·655	251·799	2·947	39·53	2·55	0·41	11·61	248·67
	12	337·756	252·284	2·974	40·00	2·58	0·42	11·73	248·84
	16	337·849	252·728	2·999	40·50	2·61	0·43	11·80	249·00
	20	337·936	253·129	3·023	41·01	2·64	0·44	11·84	249·16
	24	338·014	253·487	3·046	41·52	2·67	0·44	11·83	249·32
	28	338·083	253·800	3·069	42·05	2·71	0·45	11·78	249·48
Aug.	1	338·142	254·066	3·091	42·59	2·74	0·44	11·67	249·63
	5	338·192	254·283	3·112	43·15	2·78	0·44	11·52	249·78
	9	338·229	254·449	3·133	43·70	2·81	0·43	11·33	249·91
	13	338·255	254·563	3·152	44·26	2·85	0·41	11·07	250·04
	17	338·270	254·625	3·170	44·83	2·88	0·39	10·76	250·17
	21	338·274	254·633	3·186	45·39	2·92	0·37	10·41	250·30
	25	338·265	254·589	3·201	45·93	2·95	0·35	10·00	250·43
	29	338·244	254·493	3·215	46·47	2·99	0·32	9·54	250·58
Sept.	2	338·212	254·342	3·226	47·00	3·02	0·29	9·02	250·74
	6	338·169	254·141	3·236	47·52	3·06	0·26	8·46	250·91
	10	338·110	253·889	3·244	47·99	3·09	0·23	7·84	251·10
	14	338·041	253·588	3·249	48·43	3·12	0·20	7·17	251·35

Jupiter, 1904–5. By A. C. D. Crommelin.

Greenwich Noon.		Longitude of Ⅱ's Central Meridian. 877°·90 I.	870°·27 II.	corr. for Phase.	Light-time.	A—O.	B.
1904.		°	°	m	m	°	°
May	1	60·13	251·74	+0·11	48·542	233·96	+2·48
	5	330·96	132·05	·14	48·312		
	9	241·81	12·38	·17	48·059	234·68	2·51
	13	152·70	252·75	·19	47·783		
	17	63·61	133·14	·23	47·484	235·42	2·53
	21	334·55	13·56	·26	47·164		
	25	245·53	254·02	·29	46·824	236·16	2·55
	29	156·54	134·50	·32	46·464		
June	2	67·58	15·02	·35	46·086	236·89	2·57
	6	338·65	255·57	·39	45·690		
	10	249·76	136·16	·42	45·277	237·62	2·59
	14	160·91	16·78	·45	44·848		
	18	72·09	257·44	·48	44·405	238·35	2·61
	22	343·31	138·14	·50	43·949		
	26	254·56	18·87	·53	43·481	239·09	2·63
	30	165·86	259·64	·55	43·004		
July	4	77·20	140·45	·57	42·518	239·83	2·65
	8	348·58	21·30	·59	42·023		
	12	260·00	262·20	·60	41·523	240·55	2·67
	16	171·46	143·14	·61	41·019		
	20	82·97	24·13	·61	40·512	241·29	2·69
	24	354·52	265·16	·61	40·003		
	28	266·12	146·23	·60	39·496	242·02	2·71
Aug.	1	177·76	27·35	·59	38·993		
	5	89·45	268·52	·58	38·494	242·76	2·73
	9	1·18	149·73	·56	38·002		
	13	272·96	30·99	·53	37·521	243·49	2·75
	17	184·78	272·29	·50	37·051		
	21	96·65	153·63	·47	36·596	244·22	2·77
	25	8·56	35·02	·44	36·157		
	29	280·51	276·46	·40	35·736	244·95	2·78
Sept.	2	192·51	157·93	·35	35·337		
	6	104·54	39·44	·31	34·959	245·68	2·80
	10	16·61	280·99	·27	34·609		
	14	288·71	162·56	·22	34·286	246·42	2·81

Greenwich Noon. 1904.	P.	L−O.	B.	Apparent Diameter. Equat. Diam.	Excess over Polar.	Defect of Illum.	*d*.	Q.
	°	°	°	″	″	″	°	°
Sept. 18	337·971	253·242	3·252	48·85	3·15	0·16	6·46	251·64
22	337·893	252·855	3·253	49·24	3·17	0·13	5·72	251·98
26	337·803	252·429	3·251	49·56	3·19	0·09	4·93	252·46
30	337·708	251·969	3·246	49·84	3·21	0·06	4·10	253·17
Oct. 4	337·608	251·481	3·240	50·07	3·23	0·04	3·25	254·30
8	337·506	250·969	3·231	50·26	3·24	0·02	2·38	256·69
12	337·402	250·441	3·219	50·37	3·24	0·01	1·49	260·99
16	337·296	249·903	3·204	50·42	3·25	0·00	0·64	
20	337·192	249·361	3·188	50·41	3·25	0·00	0·47	
24	337·091	248·822	3·170	50·34	3·24	0·01	1·29	54·44
28	336·994	248·293	3·149	50·20	3·23	0·02	2·16	58·70
Nov. 1	336·901	247·780	3·126	50·00	3·22	0·04	3·04	61·65
5	336·813	247·289	3·103	49·75	3·20	0·06	3·89	64·00
9	336·731	246·826	3·079	49·45	3·18	0·08	4·71	64·88
13	336·657	246·399	3·054	49·09	3·16	0·11	5·51	65·46
17	336·591	246·010	3·028	48·70	3·14	0·14	6·26	65·90
21	336·533	245·664	3·002	48·26	3·11	0·18	6·97	66·16
25	336·482	245·365	2·977	47·77	3·08	0·21	7·64	66·40
29	336·441	245·116	2·951	47·25	3·05	0·24	8·25	66·62
Dec. 3	336·409	244·918	2·926	46·72	3·01	0·27	8·81	66·80
7	336·385	244·774	2·902	46·17	2·98	0·30	9·32	66·96
11	336·370	244·685	2·879	45·61	2·94	0·33	9·77	67·10
15	336·364	244·653	2·858	45·04	2·91	0·35	10·17	67·22
19	336·366	244·678	2·838	44·46	2·87	0·37	10·51	67 34
23	336·378	244·757	2·819	43·87	2·83	0·39	10·80	67·45
27	336·398	244·890	2·802	43·28	2·79	0·40	11·03	67·56
31	336·428	245·078	2·786	42·70	2·75	0·41	11·20	67·68
1905.								
Jan. 4	336·466	245·318	2·772	42·12	2·71	0·41	11·33	67·81
8	336·513	245·610	2·760	41·56	2·68	0·41	11·40	67·94
12	336·568	245·951	2·749	41·02	2·64	0·40	11·42	68·08
16	336·632	246·340	2·740	40·49	2·61	0·40	11·40	68·22
20	336·705	246·775	2·733	39·97	2·57	0·39	11·33	68·37
24	336·786	247·252	2·727	39·47	2·54	0·37	11·22	68·52
28	336·877	247·770	2·723	38·99	2·51	0·36	11·07	68·67
Feb. 1	336·977	248·328	2·722	38·53	2·48	0·34	10·87	68·82
5	337·085	248·923	2·720	38·09	2·45	0·33	10·64	68·97
9	337·202	249·555	2·720	37·66	2·42	0·31	10·37	69·14

Greenwich Noon.		Longitude of ☿'s Central Meridian. 877°·90 I.	870°·27 II.	corr. for Phase.	Light-time.	A—O.	B.
1904.		°	°	°	m	°	°
Sept.	18	200·84	44·16	·18	33·994		
	22	112·98	285·78	·14	33·734	247·14	2·83
	26	25·14	167·42	·11	33·510		
	30	297·32	49·08	·07	33·320	247·87	2·84
Oct.	4	209·50	290·74	·05	33·167		
	8	121·68	172·40	·03	33·051	248·60	2·86
	12	33·85	54·05	+0·01	32·978		
	16	306·01	295·69	·00	32·946	249·33	2·87
	20	218·15	177·31	·00	32·953		
	24	130·26	58·90	−0·01	33·000	250·07	2·89
	28	42·33	300·45	·02	33·091		
Nov.	1	314·36	181·96	·04	33·220	250·80	2·90
	5	226·35	63·43	·06	33·388		
	9	138·29	304·85	·10	33·593	251·54	2·91
	13	50·16	186·21	·13	33·839		
	17	321·98	67·51	·17	34·118	252·27	2·92
	21	233·74	308·75	·21	34·430		
	25	145·43	189·92	·26	34·775	253·00	2·94
	29	57·05	71·02	·30	35·149		
Dec.	3	328·60	312·05	·34	35·550	253·73	2·95
	7	240·09	193·02	·38	35·976		
	11	151·51	73·92	·41	36·424	254·45	2·96
	15	62·86	314·75	·45	36·891		
	19	334·14	195·51	·48	37·374	255·19	2·97
	23	245·36	76·21	·51	37·871		
	27	156·51	316·85	·53	38·383	255·92	2·98
	31	67·60	197·43	·55	38·903		
1905.							
Jan.	4	338·64	77·95	·56	39·430	256·65	2·99
	8	249·62	318·42	·56	39·960		
	12	160·56	198·84	·57	40·493	257·37	2·99
	16	71·45	79·21	·56	41·027		
	20	342·29	319·53	·56	41·558	258·10	3·00
	24	253·09	199·81	·55	42·085		
	28	163·85	80·06	·53	42·606	258·84	3·01
Feb.	1	74·58	320·27	·51	43·119		
	5	345·28	200·45	·49	43·621	259·56	3·02
	9	255·95	80·60	·47	44·112		

Greenwich Noon. 1904.	P.	L−O.	B.	Apparent Diameter. Equat. Diam.	Excess over Polar.	Defect of Illum.	d.	Q.
	°	°	°	″	″	″	°	°
Feb. 13	337·329	250·219	2·721	37·26	2·39	0·29	10·07	69·31
17	337·465	250·913	2·724	36·87	2·37	0·27	9·74	69·50
21	337·610	251·636	2·727	36·51	2·35	0·25	9·38	69·70
25	337·763	252·386	2·731	36·16	2·33	0·23	9·00	69·92
Mar. 1	337·926	253·161	2·736	35·84	2·31	0·21	8·59	70·15
5	338·098	253·960	2·742	35·54	2·29	0·18	8·15	70·39
9	338·279	254·781	2·749	35·26	2·27	0·16	7·69	70·65
13	338·469	255·621	2·757	34·99	2·25	0·14	7·22	70·93
17	338·668	256·480	2·765	34·75	2·23	0·12	6·73	71·25
21	338·875	257·353	2·774	34·53	2·22	0·10	6·22	71·59
25	339·091	258·241	2·783	34·33	2·21	0·09	5·69	71·97
29	339·315	259·142	2·792	34·15	2·20	0·07	5·15	72·38
Apr. 2	339·547	260·055	+2·802	33·98	2·19	0·06	4·61	72·84

The following is a list of the Greenwich Mean Times when the the illuminated disc, and the intervals between successive passages,

Date. 1904.	Passage of Zero Meridian. System I.	System II.	Intervals between Passages. I. 9^h+	II. 9^h+
May	h m	h m	m	m
1	8 11·80	2 58·99	50·623	55·802
3	9 24·91	4 38·00		
5	0 47·39	6 17·00		
7	2 0·49	7 56·00		
9	3 13·58	9 34·98		
11	4 26·68	1 18·17		
13	5 39·75	2 57·14		
15	6 52·81	4 36·11		
17	8 5·87	6 15·07	50·608	55·787
19	9 18·91	7 54·00		
21	0 41·32	9 32·92		
23	1 54·31	1 16·04		
25	3 7·30	2 54·93		
27	4 20·28	4 33·80		
29	5 33·24	6 12·66		
31	6 46·19	7 51·50		
June				
2	7 59·14	9 30·33	50·585	55·765
4	9 12·05	1 13·38		
6	0 34·36	2 52·18		

Date. 1904.	Passages of Zero Meridian. System I.	System II.	Intervals between Passages. I. 9^h+	II. 9^h+
June	h m	h m	m	m
8	1 47·24	4 30·96		
10	3 0·11	6 9·73		
12	4 12·98	7 48·49		
14	5 25·83	0 27·23		
16	6 38·67	1 10·21		
18	7 51·51	2 48·93	50·563	55·742
20	9 4·32	4 27·63		
22	0 26·55	6 6·32		
24	1 39·32	7 44·99		
26	2 52·07	9 23·63		
28	4 4·81	1 6·54		
30	5 17·54	2 45·16		
July				
2	6 30·26	4 23·77		
4	7 42·96	6 2·36	50·537	55·717
6	8 55·64	7 40·94		
8	0 17·77	9 19·49		
10	1 30·41	1 2·32		
12	2 43·01	2 40·84		
14	3 55·66	4 19·34		

Greenwich Noon.	Longitude of ♃'s Central Meridian. 877°·90 I.	870°·27 II.	corr. for Phase.	Light-time.	A—O.	B.
1904.	°	°	°	m	°	°
Feb. 13	166·59	320·73	·44	44·591	260·29	3·02
17	77·22	200·83	·41	45·056		
21	347·82	80·92	·38	45·504	261·02	3·03
25	258·41	320·99	·35	45·936		
Mar. 1	168·98	201·05	·32	46·350	261·75	3·04
5	79·54	81·09	·29	46·744		
9	350·09	321·12	·26	47·118	262·47	3·04
13	260·64	201·15	·23	47·470		
17	171·18	81·17	·19	47·798	263·21	3·05
21	81·72	321·19	·17	48·103		
25	352·26	201·21	·14	48·385	263·93	3·05
29	262·80	81·23	·12	48·644		
Apr. 2	173·34	321·26	−0·09	48·880	264·66	+3·05

adopted zero-meridians of the two systems will pass the middle of to facilitate the determination of intermediate ones :

Date. 1904.	Passage of Zero Meridian. System I.		System II.		Intervals between Passages. I. 9^h+	II. 9^h+
July	h	m	h	m	m	m
16	5	8·27	5	57·83		
18	6	20·85	7	36·30		
20	7	33·41	9	14·75	50·510	55·689
22	8	45·95	0	57·50		
24	0	7·97	2	35·93		
26	1	20·49	4	14·33		
28	2	32·98	5	52·71		
30	3	45·46	7	31·10		
Aug.						
1	4	57·92	9	9·42		
3	6	10·37	0	52·09		
5	7	22·81	2	30·40	50·483	55·661
7	8	35·22	4	8·70		
9	9	47·61	5	46·99		
11	1	9·51	7	25·25		
13	2	21·88	9	3·50		
15	3	34·23	0	46·09		
17	4	46·56	2	24·30		
19	5	58·87	4	2·49		
21	7	11·16	5	40·67	50·457	55·634

Date. 1904.	Passage of Zero Meridian. System I.		System II.		Intervals between Passages. I. 9^h+	II. 9^h+
Aug.	h	m	h	m	m	m
23	8	23·44	7	18·83		
25	9	35·70	8	56·97		
27	0	57·50	0	39·47		
29	2	9·73	2	17·58		
31	3	21·94	3	55·66		
Sept.						
2	4	34·13	5	33·73		
4	5	46·31	7	11·79		
6	6	58·47	8	49·84	50·431	55·608
8	8	10·62	0	32·27		
10	9	22·76	2	10·28		
12	0	44·40	3	48·29		
14	1	56·52	5	26·28		
16	3	8·63	7	4·26		
18	4	20·73	8	42·23		
20	5	32·82	0	24·60		
22	6	44·89	2	2·56	50·413	55·590
24	7	56·95	3	40·50		
26	9	9·00	5	18·43		
28	0	30·64	6	56·36		

Date. 1904.	Passage of Zero Meridian. System I.		System II.		Intervals between Passages. I. 9^h+	II. 9^h+
	h	m	h	m	m	m
Sept. 30	1	42·68	8	34·29		
Oct. 2	2	54·72	0	16·61		
4	4	6·75	1	54·51		
6	5	18·77	3	32·41		
8	6	30·80	5	10·32	50·406	55·583
10	7	42·83	6	48·23		
12	8	54·87	8	26·15		
14	0	16·51	0	8·48		
16	1	28·55	1	46·40		
18	2	40·60	3	24·33		
20	3	52·66	5	2·26		
22	5	4·72	6	40·20		
24	6	16·80	8	18·17	50·419	55·596
26	7	28·91	0	0·56		
28	8	41·04	1	38·56		
30	0	2·73	3	16·58		
Nov. 1	1	14·92	4	54·63		
3	2	27·10	6	32·69		
5	3	39·31	8	10·78		
7	4	51·54	9	48·90		
9	6	3·80	1	31·41	50·455	55·631
11	7	16·09	3	9·58		
13	8	28·41	4	47·78		
15	9	40·76	6	26·00		
17	1	2·66	8	4·25		
19	2	15·05	9	42·52		
21	3	27·46	1	25·16		
23	4	39·90	3	3·40		
25	5	52·38	4	41·85	50·498	55·676
27	7	4·88	6	20·24		
29	8	17·41	7	58·66		
Dec. 1	9	29·98	9	37·10		
3	0	52·05	1	19·88		
5	2	4·67	2	58·39		
7	3	17·31	4	36·92		

Date. 1904.	Passage of Zero Meridian. System I.		System II.		Intervals between Passages. I. 9^h+	II. 9^h+
	h	m	h	m	m	m
Dec. 9	4	29·98	6	15·49		
11	5	42·68	7	54·08	50·542	55·720
13	6	55·41	9	32·69		
15	8	8·17	1	15·61		
17	9	20·96	2	54·29		
19	0	43·21	4	32·98		
21	1	56·02	6	11·65		
23	3	8·88	7	50·46		
25	4	21·77	9	29·24		
27	5	34·70	1	12·28	50·584	55·762
29	6	47·63	2	51·10		
31	8	0·58	4	29·95		
1905. Jan. 2	9	13·55	6	8·80		
4	0	35·95	7	47·70		
6	1	48·97	9	26·60		
8	3	2·01	1	9·74		
10	4	15·07	2	48·69		
12	5	28·14	4	27·66	50·615	55·795
14	6	41·22	6	6·64		
16	7	54·32	7	45·65		
18	9	7·44	9	24·66		
20	0	29·95	1	7·90		
22	1	43·12	2	46·95		
24	2	56·30	4	26·02		
26	4	9·49	6	5·10		
28	5	22·69	7	44·19	50·641	55·820
30	9	35·90	9	23·29		
Feb. 1	7	49·13	1	6·60		
3	9	2·37	2	45·74		
5	0	24·96	4	24·88		
7	1	38·22	6	4·04		
9	2	51·48	7	43·21		
11	4	4·76	9	22·39		
13	5	18·05	1	5·72	50·657	55·837
15	6	31·35	2	44·90		
17	7	44·65	4	24·09		

Date. 1904.	Passage of Zero Meridian. System I.		System II.		Intervals between Passages. I. 9^h+	II. 9^h+
Feb.	h	m	h	m	m	m
19	8	57·96	6	3·38		
21	0	20·61	7	42·58		
23	1	33·94	9	21·79		
25	2	47·26	1	5·15		
27	4	0·60	2	44·37		
Mar.						
1	5	13·94	4	23·61	50·669	55·848
3	6	27·29	6	2·85		
5	7	40·64	7	42·10		
7	8	53·99	9	21·35		
9	0	16·68	1	4·76		
11	1	30·04	2	44·02		
13	2	43·40	4	23·28		
15	3	56·76	6	2·55		
17	5	10·12	7	41·82	50·673	55·854
19	6	23·49	9	21·08		
21	7	36·86	1	4·50		
23	8	50·23	2	43·77		
25	0	12·93	4	23·04		
27	1	26·30	6	2·31		
29	2	39·67	7	41·58		
31	3	53·04	9	20·85		
Apr.						
2	5	6·41	1	4·27	50·674	55·854

The quantities in the ephemeris are to be interpolated directly for the times for which they are required, the equation of light having been already applied.

The position of *Jupiter's* North Pole is assumed to be R.A. $17^h\ 51^m\ 58^s·89$, N.P.D. 25° 26′ 13″·5 at the beginning of 1904, and R.A. $17^h\ 51^m\ 59^s·13$, N.P.D. 25° 26′ 14″·1 at the beginning of 1905.

P denotes the position-angle of the northern extremity of *Jupiter's* axis, reckoned eastward from the northernmost point of the disc.

L−O+180°, Λ−O+180° are the jovicentric right ascensions of the Earth and Sun respectively, reckoned in the plane of the planet's equator from O, the point of the vernal equinox of *Jupiter's* northern hemisphere; B, *B* are the jovicentric declinations of the Earth and Sun above the planet's equator.

The adopted values of the diameters at distance 5·20 are: Equatorial 38″·419; Polar 35″·945.

The assumed time for light to traverse the unit distance is $498^s·92$, this being the same value as that used by Mr. Marth.

d denotes the jovicentric angle between the Earth and Sun.

Q denotes the position-angle of the point of greatest phase, and is reckoned eastward from the northernmost point of the disc. It also gives the position-angle of the shadows of the satellites measured from the satellites themselves.

If we call B″ the jovigraphical latitude of the centre of the disc, then we can find B″ by the formula:

$$B'' = \sec^2 \varepsilon_0\, B, \text{ where } \sec \varepsilon_0 = \frac{a}{b} = \frac{15·53}{14·53}.$$

The longitudes of *Jupiter's* central meridian are computed with unaltered values of the rates of rotation and of the zero-meridians in the two adopted systems. The addition of the

"Corr. for Phase" gives the longitudes of the meridians which bisect the illuminated disc.

The sidereal periods of rotation corresponding to the two adopted systems are $9^h\ 50^m\ 30^s{\cdot}004$, $9^h\ 55^m\ 40^s{\cdot}632$.

The ephemeris has been somewhat abbreviated as compared with those in recent years, being for every four days instead of every two days. Instead of giving every transit of the zero meridian, only one in every two days is given; but any intermediate transit may be readily found by applying to the nearest transit in the table once or twice the interval between successive passages, which interval is tabulated every sixteen days.

I have received several observations of the longitude of the Great Red Spot in System II. from Mr. W. F. Denning and Rev. T. E. R. Phillips, from which I have deduced the following mean values.

Date. 1903.	Longitude. °	No. of Observations.	Date. 1903.	Longitude. °	No. of Observations.
May 28	30·0	2	Sept. 8	32·9	5
June 26	32·0	3	Oct. 23	34·6	4
July 11	31·4	5	Nov. 19	34·4	6
Aug. 14	32·7	8	Dec. 7	34·4	2*

It appears that the rapid diminution in the longitude which took place in 1902 has ceased and been followed by a slow recovery. Rev. T. E. R. Phillips considers that the spot was more conspicuous in 1903 than it has been for some years; it should therefore be carefully watched in case of a revival of activity.

A list of times of elongation of the fifth satellite is given in the *Connaissance des Temps*.

It may be mentioned here that the *Connaissance des Temps* for 1899 and following years gives ephemerides for the satellites of *Mars*, *Saturn*, *Uranus*, and *Neptune* in the same form as those formerly contributed to the *Monthly Notices* by Mr. Marth.

* To these may be added the mean of two observations just received from Mr. Denning, which give 35°·3 on 1904 January 8.

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1904 *January* 2.